Question 1.

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Introduction

The Black-Scholes Model is a famous option pricing formula and was first introduced by two financial economists Fischer Black and Myron Scholes in the Journal of Political Economy in 1973. It has been proved to be a significant innovation in the area of financial pricing.

The Formula

The Nobel formula is written as:

$$C = S_0 N(d_1) - Ee^{-\rho T} N(d_2)$$

$where:$

$$d_1 = \frac{\ln(S_0 / E) + (\rho + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Generally, the basic option valuation method is to divide the option value into two parts – intrinsic value and time value of the option. For instance, the intrinsic value of a stock call option is the underlying stock price minus the exercise price ($S_0 - E$). But the exercise should be adjusted by the time value of the money. Then, the “adjusted” intrinsic value is the stock price minus the present value of the exercise price

($S_0 - PV(E)$). As can be seen from the formula above, $S_0 - Ee^{-\rho T}$ is the intrinsic value. The time value of an option can be seen as some kind of volatility value which is implied in $N(d_1)$ and $N(d_2)$ of the formula above. The terms $N(d_1)$ and $N(d_2)$ serve as risk-adjusted probabilities of S_0 and $Ee^{-\rho T}$ respectively.

Assumptions

The Black-Scholes Pricing model, as other models, is based on a specific set of assumptions:

Firstly, the underlying stock prices (S_0) are assumed to be random and stock returns are lognormally distributed. As all known, although the portfolio managers or other market participants may catch some stock returns' long-term tendencies, stock returns are almost unpredictable, especially on the short run. Moreover, many empirical studies show that log returns are distributed the way close to the normal distribution; hence, the assumption is reasonable.

Secondly, the risk-free rate (ρ) and volatility of the log return on the underlying stock are assumed to be constant (homoskedasticity). This assumption is always not the case in real world. The interest rate does not remain constant. In addition, the assumption of constant volatility is always violated in reality. More complex models are needed if the assumption of constant volatility is relaxed.

Thirdly, no taxes or transaction costs are considered. Most financial pricing models are based on this assumption. But unfortunately, taxes and transaction costs do have an impact on the options' pricing. There is advanced methods to relax this assumption.

Fourthly, the underlying stock pays no dividends. Most stocks pay dividends. But this assumption can be relaxed by subtracting a discrete dividend adjusted term from the underlying stock price (S_0) or dividing a continuous dividend yield adjusted term from the underlying stock price (S_0).

Finally, the options are European options. Black-Scholes model cannot price early exercise options, whereas the binomial model can do. However, Black-Scholes model provide a way to better understand how the early exercise works.

Empirical evidence on option pricing

There have been plenty of empirical studies relating to the accuracy or otherwise of the Black-Scholes option pricing model. Most of these studies have been positive because the results generated from this pricing model are fairly close to the actual trading prices. The model does guide the market participants to trade on options. On the other hand, some papers also began to doubt the reliability of

this accepted valuation model in practice. Commonly, there are several criteria to accept or reject a model.

The first criterion is to see whether the valuations from the application of the model conform to reality or are close to the actual trading prices and whether the assumptions can be met.

Some studies show that the Black-Scholes model seems to perform poorly for valuing deep in the money calls and deep out of the money calls. Deep in the money calls are always undervalued and deep out of the money calls are overvalued. Other studies focus on the reality of the assumptions. Geske and Roll[1] have pointed out that the reason of the observed empirical mispricing by Black-Scholes model is that the model fails to consider two factors - the early exercise possibility and that the underlying stocks pay dividends.

According to the assumptions, the volatility implied by the Black-Scholes option model with different exercise prices (*E*) and the time to maturity (*T*) should remain constant over time. And the implied volatility should be consistent with the actual volatility observed in the underlying stock market. But this hypothesis is always violated in real world. Rubinstein[2] has argued that the implied volatility does not stay constant. It is a decreasing function of exercise prices and can be an increasing

[1] Robert Geske and Richard Roll, "on Valuing American Call Options with the Black-Scholes European Formula", *Journal of Finance* 39 (June 1984).
[2] Mark Rubinstein, "Implied Binomial Trees", *Journal of Finance* 49 (July 1994).

or decreasing function of the time to maturity. He also found that the volatility implied by the Black-Scholes option model is different from that observed directly from underlying stocks.

Whether there is a better model could be the second criterion. There are two basic option valuation models – binomial option pricing model and Black-Scholes option pricing model. The binomial option pricing model is simple in its formula term. It allows the underlying stock price to go either up or down, possibly at different rates which follow a binomial probability distribution. That the probabilities to go up or down are available and constant over time and that risk free rate is constant over time are assumed in the binomial model. Unlike the Black-Scholes model, it can value early exercise American options and the options with the underlying stocks which pay dividends. Although the binomial option pricing model is more flexible and simple in its formula term, it requires a computer program to run the complex algorithm involved. The binomial model is a discrete time model whereas the Black-Scholes model is a continuous time model. The results generated from the binomial model converge to that of the Black-Scholes model as the number of time intervals increases. Both models are important and widely used in practice.

There are numerous advanced models emerging in the recent financial literatures. However, most of them come from the basic structure as the Black-Scholes model. GARCH model (Generalized Autoregressive Conditional

Heteroskedasticity) has gained widespread acceptance in the literature. The model relaxes the critical assumption of the Black-Scholes model – homoskedasticity, however, it become much more complex and not widely applied in practice.

The third criterion is whether the model is widely used in practice. There is no doubt that the Black-Scholes model is widely applied in practice and in the literature as well. Most market participants apply this model to value options, hedge their portfolio positions and generate implied volatilities.

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