ECON G4S
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RISK, UNCERTAINTY AND INFORMATION

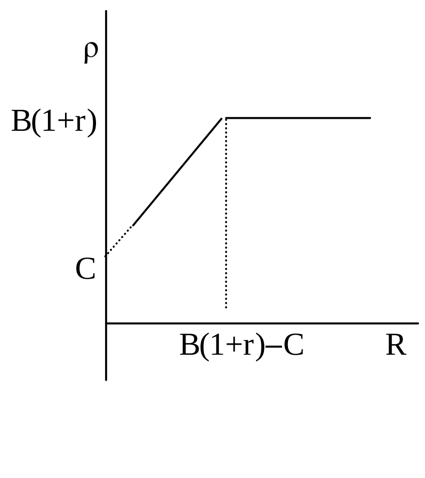
LECTURE 5 CREDIT RATIONING

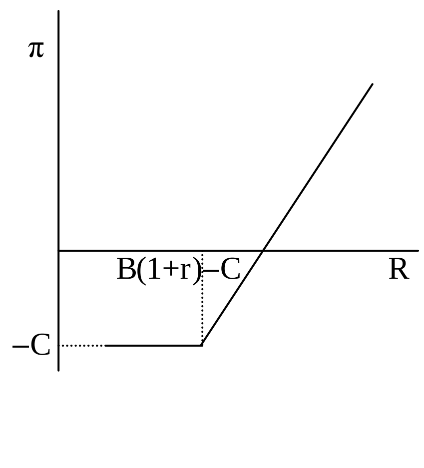
Credit rationing exists when some borrowers cannot obtain loans, even though indistinguishable from other borrowers who do obtain loans. The rate of interest (which differs from a lender's expected rate of return because of the risk of default) influences the quality of a loan, by acting as: (a) a screening device; (b) an incentive device. "Both effects derive directly from the residual imperfect information which is present in loan markets after banks have evaluated loan applications." (Stiglitz and Weiss) Thus, for lenders, such as banks, there may be an optimal rate of interest. When there is, credit rationing may exist, even in equilibrium. The lender does not respond to excess demand for loans by raising the rate of interest above its optimal level.

1. Model 1

A lender is faced by borrowers, indexed by θ. A borrower who obtains a loan invests in a specific project. All projects have the same mean return, but risk increases with θ. The (gross) return on a project, denoted by R, is uniformly distributed between k and s, where s-k increases with θ. (The increase in s-k is an example of an increase in 'mean preserving spread' and implies an increase in R's variance.) The amount of a loan is B, the rate of interest r and collateral C. There is limited liability. The lender's and borrower's returns are, respectively:

$$\rho(r,\theta) = \min\,[B(1+r),\ C+R]$$
$$\pi(r,\theta) = \max\,[R - B(1+r),\ -C].$$

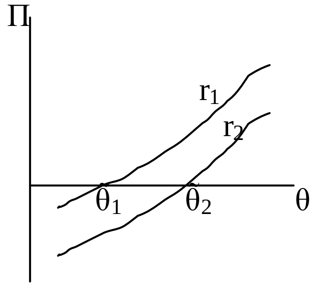




The borrower's expected return is:

$$\Pi = \frac{1}{s-k}\int_k^s \max[R - B(1+r), -C]dR.$$

Π is decreasing in r and, since π(r,θ) is convex, increasing in θ. Suppose, for given r, Π=0 when $\theta=\tilde{\theta}$. Borrowers for whom $\theta<\tilde{\theta}$ find loans unprofitable and exit the market. Borrowers for whom $\theta>\tilde{\theta}$ find loans profitable and remain in the market . As r increases , the threshold for borrowers , $\tilde{\theta}$, also increases.



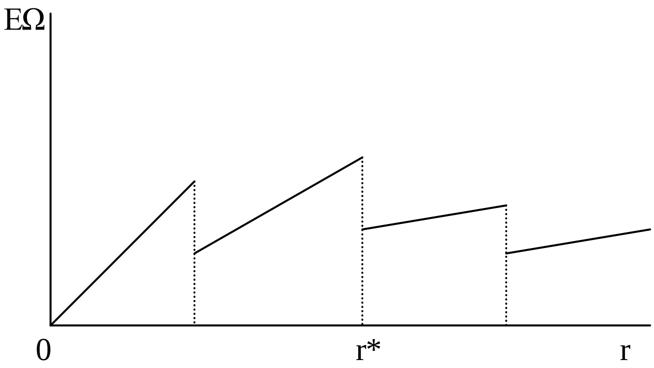
$\tilde{\theta}$ increases from $\tilde{\theta}_1$ to $\tilde{\theta}_2$ when r increases from r_1 to r_2.

The lender's expected return is:

$$\Omega = \frac{1}{s-k}\int_k^s \min[B(1+r), C+R]dR.$$

Ω is increasing in r and, since ρ(r,θ) is concave, decreasing in θ.

Suppose there are several distinct classes of borrower . As r increases , $\tilde{\theta}$ increases. First, the class with the safest projects leaves the market, then the class with the next safest, etc. When a class exits the market, EΩ falls. Classes with riskier projects remain in the market, i.e. increasing r causes adverse selection. Thus, typically, there exists a value for r, r=r*, that maximizes the expected return to the lender, averaged over borrowers. This optimal value for r can be an equilibrium rate.

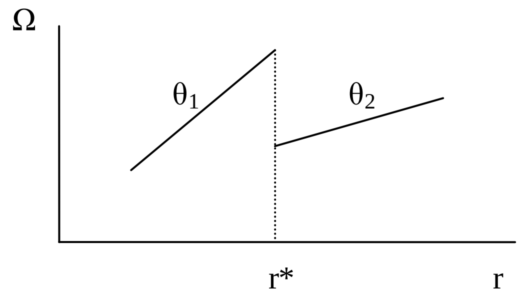


2. Model 2

Consider next a borrower who can choose either a safe or a risky project. Denote these by θ_1 and θ_2. The return to the safe project is $\tilde{R}$, and the return, R, to the risky project, which has a lower mean, is uniformly distributed between k and s. The amount borrowed is B, the rate of interest r and collateral C. As r increases ($1+r < \tilde{R}$), the borrower initially chooses θ_1. However, at some critical value for r, r=r*, the two projects yield the same expected return to the borrower. At higher levels of r, he or she chooses the risky project. r* is determined by:

$$\tilde{R} - B(1+r^*)$$
$$= \frac{1}{s-k}\int_k^s \max[R - B(1+r^*), -C]\,dR.$$

Due to moral hazard, the expected return to the lender, Ω, falls as r crosses the threshold, r=r*. Thus, r* can be the lender's optimal rate of interest (see diagram). Again there can be credit rationing in equilibrium.



3. Model 3

Suppose a mix of risk-averse and risk-neutral borrowers. Again, all projects have the same mean return. Risk-averse borrowers choose safe projects and risk-neutral borrowers risky projects. As the rate of interest r increases, increasing numbers of risk-averse borrowers exit the market, thus raising the proportion of risk-neutral borrowers. This means that, whereas the expected return to the lender on a loan increases with r, the average expected return on all loans starts at some point to fall, due to adverse selection. I.e. there exists r=r* which is optimal for the lender.

With this model, there can also be an optimal level for C, the amount of collateral required. As C increases, highly risk-averse borrowers are not much affected. Some borrowers who are less risk averse or risk neutral are screened out, as risky projects put collateral at risk. Others, however, adopt more and more risky projects. Thus, as well as screening effects, we have moral hazard effects.

4. Model 4

A bank lends to a farmer. The amount of the loan is B, the rate of interest r and collateral C. The (gross) return, depending on both the farmer's effort (e) and the weather, is R. There are two possible outcomes, R=0 and R=S, with probabilities 1-p and p. S is fixed, but p=p(e), with p′(e)>0 and p″(e)<0. I.e. greater effort increases the probability of success. Effort, however, is not contractible.

The farmer's profits are:

$$\pi = \max[R-B(1+r), -C].$$

The farmer's utility function is:

$$\begin{aligned} u &= \pi - e \\ &= \max[R-B(1+r), -C] - e. \end{aligned}$$

Taking expectations:

$$\begin{aligned} Eu &= p[S-B(1+r)] + (1-p)(-C) - e \\ &= p(e)[S-B(1+r)+C] - C - e. \end{aligned}$$

Thus the optimal level of effort, e*, is given by the first order condition:

(1) $$p'(e^*) = \frac{1}{S - B(1+r) + C}.$$

The *debt overhang* problem is the reduction of e*, due to moral hazard, below its 'first-best' level, which is given by:

$$p'(e^*) = \frac{1}{S}.$$

An increase in r reduces e*. To show this formally, differentiate (1) with respect to r:

$$p''(e^*)\frac{de^*}{dr} = \frac{B}{[S - B(1+r) + C]^2}$$

(2) $$\frac{de^*}{dr} = \frac{[p'(e^*)]^2 B}{p''(e^*)}.$$

The bank's (gross) return is:

$$\rho = \min[B(1+r), C+R].$$

Let the bank obtain loanable funds at the rate s. The bank's expected profit is:

(3) $$\begin{aligned} \Omega &= pB(1+r) + (1-p)C - B(1+s) \\ &= p[B(1+r) - C] + C - B(1+s). \end{aligned}$$

The optimal rate of interest, r*, is given by the first order condition:

$$p'(e^*)\frac{de^*}{dr}[B(1+r^*) - C] + p(e^*)B = 0.$$

Substituting from (2):

(4) $$B(1+r^*) - C = -\frac{p''(e^*)p(e^*)}{[p'(e^*)]^3},$$

where e* is determined by (1).

The optimal rate of interest, r*, balances the higher returns to the bank when the farm is successful, obtained by increasing r, against the corresponding lower effort, e*, by the farmer, and reduced probability of success. The existence of r* indicates 'macro rationing'.

5. Model 5

Now suppose S depends on the amount borrowed, i.e. S=S(B), where S'>0 and S"<0. The farmer's demand for loans is given by the first order condition:

(5) $$S'(B) = 1+r.$$

Differentiate (1) with respect to B:

$$p''(e^*)\frac{de^*}{dB} = \frac{1+r-S'(B)}{[S-B(1+r)+C]^2}$$

(6) $$\frac{de^*}{dB} = \frac{[p'(e^*)]^2[1+r-S'(B)]}{p''(e^*)}.$$

Using (3) and (6), the bank's supply of loans, B=B*, is given by the first order condition:

$$p'(e^*)\frac{de^*}{dB}[B(1+r^*)-C] + p(e^*)(1+r^*)-(1+s) = 0$$

(7) $$B^*(1+r^*)-C = -\frac{p''(e^*)[p(e^*)(1+r^*)-(1+s)]}{[p'(e^*)]^3[1+r^*-S'(B^*)]}.$$

Comparing (4) and (7), we obtain:

(8) $$S'(B^*) = \frac{1+s}{p(e^*)}.$$

From (5) and (8), 'micro rationing' exists, i.e. the farmer's demand for loans greater than the bank's supply, if:

(9) $$1+r < \frac{1+s}{p(e^*)}.$$

From (3), (9) is satisfied if, in particular, Ω=0 and C>0, i.e. if by the zero profit theorem the bank's expected profit is zero and collateral is positive.

<u>Reading:</u>

Stiglitz, J E and A Weiss, "Credit rationing in markets with imperfect competition", *The American Economic Review* 71,
393-410, 1981.
Stiglitz, J, "The causes and consequences of the dependence of quality on price", *Journal of Economic Literature* 25, 1-48.
Ghosh, P, D Mookherjee and D Ray, "Credit rationing in developing countries: An overview of the theory", in Mookherjee, D and D Ray (eds.), *A Reader in Development Economics*, Blackwell 2000.