Seminar 6: Addendum

The GPH regression for lnr (log real exchange rate) indicated that the series is covariance nonstationary $(d \geq 0.5)$. However, the GPH regression requires $d < 0.5$ (i.e., covariance stationarity) for consistency and asymptotic normality. Therefore we will repeat the GPH regression for the first difference of the log real exchange rate (the real exchange rate returns). These returns will be stationary and hence will give statistically more reliable estimates of the long memory parameter.

Note that if a series $y_t \sim I(d)$ then $(1-L)y_t \sim I(d-1)$ (in words: the first difference is integrated of order $d-1$). So if $0.5 \leq d < 1.5$ (i.e., y_t is covariance nonstationary) then $(1-L)y_t$ will be stationary: it will be integrated of some order in the interval $-0.5 \leq d-1 < 0.5$.

Based on the above, the GPH test for the real exchange rate returns will provide an estimate of $d-1$. Importantly, since the returns are stationary $(d-1 < 0.5)$, the GPH regression will be consistent and asymptotically normal.

Firstly, with the 'untitled page' (first page) in the workfile active, create the real exchange rate returns:

On the workfile toolbar click ***Genr*** and enter dlnr=d(lnr)

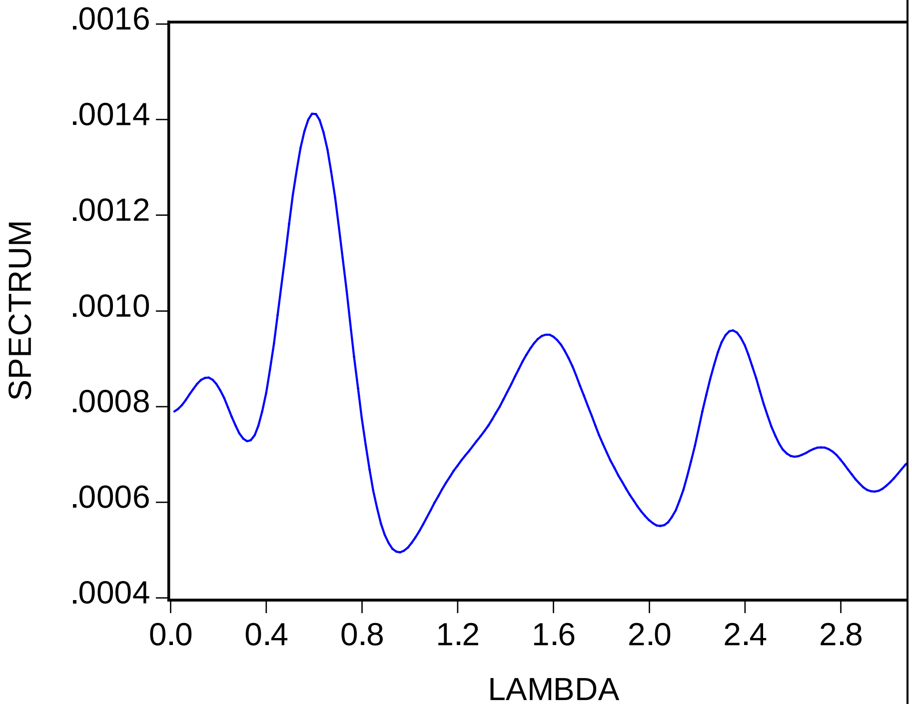
Now estimate the spectrum for dlnr

Open spectrum.prg and click on the ***Run*** button.

Program Arguments: **dlnr 20**

Execution mode: **Quiet**

Figure A1: Spectrum of the real exchange rate returns



Compare this spectrum with the previous one for the log level of the series. First differencing has removed the spike at frequency zero. In other words differencing has removed the long-run trend component from the series.

In particular, the spectrum appears to have a finite positive value at frequency zero. This suggests that the returns are I(0) (and hence the log real exchange rate itself is I(1)). See box below: ‘Different shapes of spectra around frequency zero’

Different shapes of spectra around frequency zero

Consider the process $y_t = (1-L)^{-d}\,\varepsilon_t$. This process is integrated of order d : $I(d)$. For example: if $d = 1$ this is a <u>random walk/martingale process</u> $(I(1))$; if $d = 0$ this is a <u>white noise process</u> $(I(0))$; if d is a fractional value then this is a <u>fractional white noise process</u> $(I(d))$.

The spectrum for this process is given by (see lecture 7):

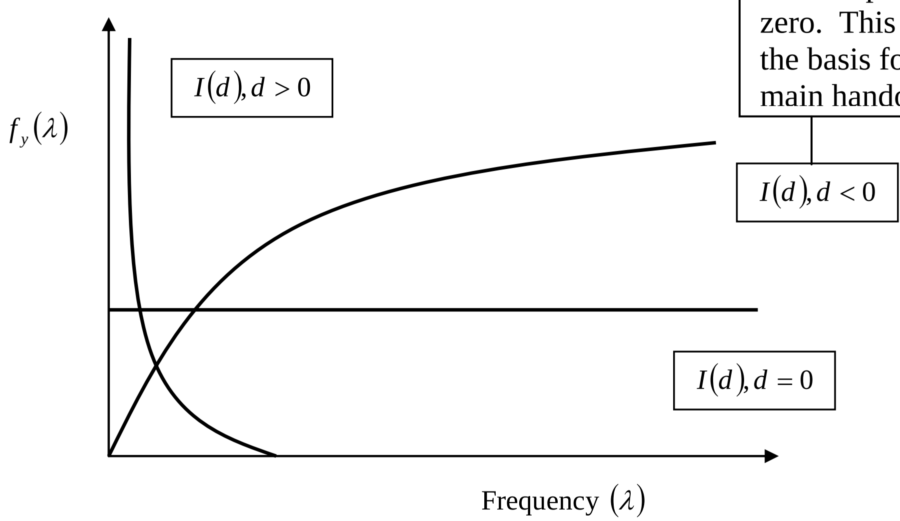
$$f_y(\lambda) = \left|1 - e^{-i\lambda}\right|^{-2d}\,\sigma^2/2\pi$$

- For $d > 0$ the spectrum has a spike (<u>escapes to infinity</u>) at frequency zero:
  - $f_y(0) = |1-1|^{-2d}\,\sigma^2/2\pi = \infty$.
- For $d = 0$ the spectrum has a <u>finite positive value</u> at frequency zero
  - $f_y(0) = |1-1|^{0}\,\sigma^2/2\pi = \sigma^2/2\pi$.
- For $d < 0$ the spectrum is <u>zero</u> at frequency zero:
  - $f_y(0) = |1-1|^{-2d}\,\sigma^2/2\pi = 0$

$-2d > 0$ for $d < 0$

For example, <u>over differencing</u> an $I(0)$ process results in an $I(-1)$ process which has a zero valued spectrum at frequency zero. This observation forms the basis for the KPSS test (see main handout for Seminar 6).

Figure A2: Shapes of spectra around frequency zero



Finally run the GPH regression using the returns spectrum to estimate $d-1$.

On the main toolbar click ***Quick/Estimate Equation*** and enter:

log(spectrum) c log(4*sin(lambda/2)^2)

Sample: **1 20**

Dependent Variable: LOG(SPECTRUM)
Method: Least Squares
Date: 03/08/07 Time: 19:11
Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-7.152723	0.032983	-216.862	0
LOG(4*SIN(LAMBDA/2)^2)	-0.007948	0.0076	-1.04576	0.3095

R-squared	0.057276	Mean dependent va	-7.12
Adjusted R-squared	0.004903	S.D. dependent var	0.05
S.E. of regression	0.05377	Akaike info criterion	-2.91
Sum squared resid	0.052042	Schwarz criterion	-2.81
Log likelihood	31.13566	F-statistic	1.09
Durbin-Watson stat	0.09347	Prob(F-statistic)	0.30

$\hat{d}-1=-\hat{\beta}=0.00795$. This estimate is insignificantly different from zero indicating that $d=1$. In other words the log real exchange rate is $I(1)$.

<u>In this case the ADF, P-P, KPSS and GPH tests are all telling the same story: the log real exchange rate is $I(1)$ and PPP does not hold in the sample.</u>