# Uncertainty and Competition in the Adoption of Complementary Technologies

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## Abstract

In this paper we study, for a duopoly market, the combined effect of uncertainty, competition and "technological complementarity" on firms' investment behaviour for a game-choice setting where it is assumed that there is a first-mover advantage. Firms do often use inputs whose qualities are complements such as computer and modem, equipment and structure, train and track, and transmitter and receiver and, therefore, on such cases, investment decisions on upgrades or replacements must consider the degree of complementarity between investments.

We derive analytical or quasi-analytical solutions for the leader and the follower value functions and their respective investment thresholds. At the beginning of the investment game firms have two technologies available, whose functions are complement, and the option to adopt both technologies at the same time or at different times, in a context where the evolution of the gains that can be made through the adoption of the technology(ies) and the price of the technologies are uncertain.

Our results contradict the conventional wisdom which says that "when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously". We found that when uncertainty about revenues and the price of the two technologies is considered it might be optimal for the leader and the follower to adopt the two technologies asynchronously, first, the technology whose price is decreasing at a lower rate and then the technology whose price is decreasing more rapidly. Some of the illustrated results show nonlinear and complex investment criteria and sensitivities to the expected rate of change in the price of the technologies and the degree of complementarity between the two technologies.

## 1. Introduction

Since the pioneering work of Smets (1993) the effect of uncertainty and competition on firms' investment behaviour has been extensively studied in the real options literature[3], but the influence of the degree of complementarity between the inputs of an investment on firms' investment decisions has been neglected. However, firms do often use inputs whose qualities are complements, such as computer and modem, equipment and structure, train and track, and transmitter and receiver. In such cases, investment decisions on upgrades or replacements must consider the degree of complementarity between investments.

Conventional wisdom says that when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously, i.e., when raising the quality of one input it should upgrade its complements at the same time[4]. However, this conclusion has been made for economic contexts where it is assumed that uncertainty, competition and adjustment costs are absent. In this paper we study the effect of the complementarity between two technologies on their optimal time of adoption, considering competition between firms and uncertainty about revenues and investment costs. For simplicity we neglect adjustment costs.

Our initial intuition is that when uncertainty is added to the investment problem, for instance uncertainty about the cost of the technologies, the conventional wisdom stated above may not hold, since due to technological progress the cost of a technology can decline rapidly and, therefore, when firms anticipate that the cost of technologies may not fall at the same rate, it may pay to adopt first the technology whose price is falling more slowly and wait to adopt the technologies whose price is falling more rapidly.

The concept of the complementarity between the elements of a (technological) "system" is studied here from the adopter's point of view. Our aim is to investigate to what extent the degree of complementarity between two technologies affects the adopter's investment behavior, in economic contexts where uncertainty and competition hold as well. However, the phenomenon of complementarity between the inputs of a "system" affects many other areas of the economy and, therefore, can be studied from other perspectives. For instance, it has been argued that the pace of modernization of an industry is quite often influenced by the degree of technological

[3] Dixit and Pindyck (1994), chapter 9, Grenadier (1996), Lambrecht and Perraudin (1997), Huisman (2001), Weeds (2002) and Paxson and Pinto (2005) address such problems.
[4] See Milgrom and Roberts (1990, 1995) and Javanovic and Stolyarov (2000).

complementarity between it and those whose activities are complements. This phenomenon was studied by Smith and Weil (2005) who investigated how changes in retailing and manufacturing industries, together, affected the diffusion of new information technologies in the U.S. apparel industry between 1988 and 1992. They show that the process occurs in a stepwise fashion[5], i.e., retailers typically adopted the new information technology systems first and the increased demand for rapid replenishment by retailers then stimulated suppliers to adopt new manufacturing practices and make greater investments in complementary information technologies, causing a "ratchet-up" process as the payoffs to adopting increased when more customers and suppliers, respectively, adopted. This case constitutes, according to the authors, an extraordinary example of the effect of the complementarity between new technologies on the pace of modernization of interlinked industries.

Other area where the concept of "complementarity" plays also an important role is the area of research and development (R&D), in the sense that firms, when planning their R&D activities, do make strategic decisions regarding the degree of complementarity (sometimes called compatibility) between the new products they aim to launch in the future and the complement products that are already available in the market and those they conjecture will be launched by their opponents in the future[6]. We find in the market two distinct types of behaviour (strategies) on this regard: firms who do not have a dominant market position and want to growth tend to guide their R&D efforts in order to launch new products that are, as much as possible, complement (compatible) of those from their opponents who have a dominant market positions; firms who have a dominant market position tend to guide their R&D efforts in order to launch new products that are, as much as possible, not complements (compatible) of those from their opponents. An example of the later strategy is the nine-year battle between the European Union (EU) commission and Microsoft that culminated last October 2007 with a fine of € 497 million due to its supposed misconduct in developing software that does not allow open-source software developers access to inter-operability information for work-group servers used by businesses and other big organizations[7,8].

Mamer, et al. (1987) studied the effect on investment behaviour of technological complementarity and competition between firms. They derived a model where competition is modeled in a game

[5] For a detailed description of how the new information technologies adoptions occurred in both industries see Smith and Weil (2005), pp. 494-495.

[6] Note that the diffusion of an innovation depends, to some extent, on the diffusion of complement innovations, which amplify its value.

[7] See Etro (2007), p. 221, and Financial Times, October 23, 2007, p. 1.

[8] Note that Microsoft has 95 per cent market share in desktop publishing and more than 70 per cent of work-group server operating systems.

theoretic setting and show that firms' optimal investment strategy, when uncertainty about the profitability of the innovation is unknown and competition is considered, is characterized by a monotone sequence of pairs of threshold values which delineate a cone-shaped continuation region. Milgrom and Roberts (1990), in an attempt to improve the understanding of the effect of the technological complementarity on the manufacturing modernization, derived an optimizing model of the firm that generates many of the observed patterns that mark modern manufacturing.

Milgrom and Roberts (1995b) use the theories of super-modular optimization and games as a framework for the analysis of systems marked by complementarity, and suggest that the ideas of complementarity and super-modularity in optimization and games can be quite useful to understand the relation between strategy and organizational structure. Colombo and Mosconi (1995) study the diffusion of Flexible Automation production and design/engineering technologies in the Italian metalworking industry, giving particular attention to the role of the technological complementarity and learning effects associated with experience of previously available technologies.

The concept of "complementarity" was also used by Milgrom and Roberts (1995a), studying the Japanese economy between 1940 and 1995, in an attempt to interpret the characteristic features of Japanese economic organization in terms of the complementarity between some of the most important elements of its economic structure. Jovanovic and Stolyarov (2000) study the combined effect of complementary and non-convex cost of adjustment in the upgrade (or adoption) of new technologies, and show that if upgrading each input involves a fixed cost, the firm may upgrade them at different dates and conclude that their results might be an explanation for why investment in structures is more spiked than equipment investment, and why plants have spare capacity.

We use the real options methodology to derive, for a duopoly market and in a game-choice setting where it is assumed that there is a first-mover advantage, analytical expressions for the value functions of the leader and the follower and their respective investment trigger values. In our investment problem we assume that the market is composed of two idle firms and that at the beginning of the investment game there are two new (complementary) technologies available, *tech 1* and *tech 2*. Firms are allowed to invest twice[9], in *tech 1* and in *tech 2*; the costs firms can save due to the adoption(s) as well as the amount they pay for each technology at the time of the adoption are uncertain. More specifically, we assume that the firms' cost savings are a proportion of the market

[9] Note that, in the adoption of two new technologies, if there is uncertainty about the revenues from the adoption and the cost of the technologies, before adoption firms have the option to adopt either one or both technologies, at the same time or at different times.

revenues and that both market revenues and the cost of each technology follow independent, and possible correlated, geometric Brownian motion (gBm) processes.

The word "complementarity" between the two technologies means here the degree to which two technologies are better off when operating together rather than operating alone; $\gamma_{12}$ in inequality (10), $\gamma_{12} > \gamma_1 + \gamma_2$, is the parameter that represents the degree of complementarity between the two technologies, where, $\gamma_1$ and $\gamma_2$ are the proportion of the firm's revenues that are expected to be saved if *tech 1* and *tech 2*, respectively, are adopted alone (firms operate with just one technology), and $\gamma_{12}$ is the proportion of the firm's revenues that are expected to be saved if both technologies are adopted together (firms operate with the two technologies).

The methodology used to set the investment game is similar to that developed by Smets (1993), and followed by Dixit and Pindyck (1994), chapter 9, and Huisman (2001).

The rest of this paper is organized as follows. In section 2, we outline the model assumptions and define the duopoly investment game. In section 3, we derive the firms' value functions and their investment trigger values. In section 4, we present the results and in section 5 we conclude and give some guidelines for possible extensions of this research.

## 2. The Investment Game

The investment game is characterized as follows: in a risk neutral world, there are two idle firms studying the possibility of entering in a market by adopting one or two new technologies, *tech 1* and/or *tech 2*, at the same time or at different times, for which they have to spend a sunk (and uncertain) cost $I_1^*(t)$ and/or $I_2^*(t)$, respectively. The two technologies are currently available and the firms' cost savings that are expected to be made through the adoption of the technologies are a proportion of their revenues, whose evolution is uncertain. The two firms are allowed to invest twice, in *tech 1* and in *tech 2*, the life of each technology is assumed to be infinite and time is continuous. In Figure 1 we represent the investment game using an extensive-form representation[10].

[10] For a detailed description of this type of game representation see Gibbons (1992).

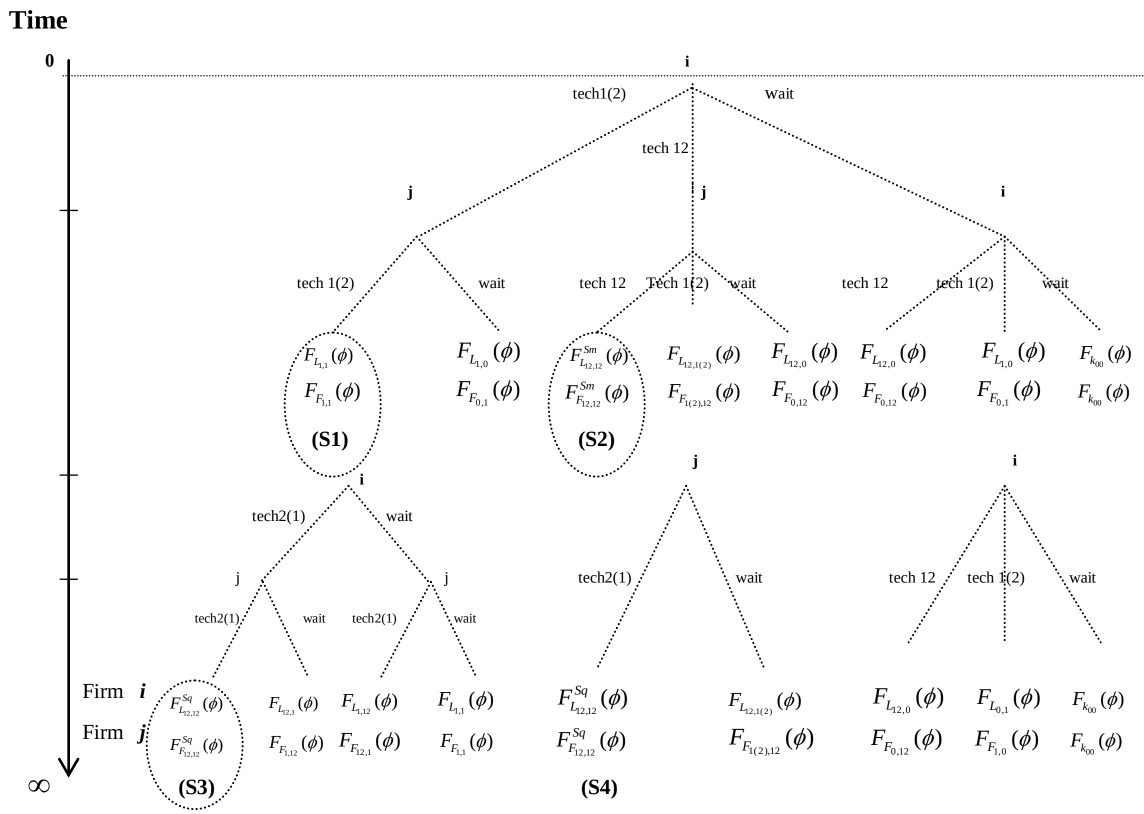


**Figure 1 -** Extensive-form representation of a Continuous Time Real Option Game (CTROG) with two firms and two complementary technologies.

Below we characterize four of the investment game scenarios described in Figure 1:

***Scenario 1* (S1):** firm *i* adopts first *tech 1(2)* and becomes the leader, firm *j* adopts later *tech 1(2)*, and becomes the follower. The payoffs for firm *i* and *j* are given, respectively, by $F_{L_{1,1}}(\phi)$ and $F_{F_{1,1}}(\phi)$. ***Scenario 2* (S2):** firm *i* adopts first *tech 1* and *tech 2* (*tech 12*) simultaneously, and firm *j* does the same later. Firm *i* becomes the leader and firm *j* the follower and their payoffs are, respectively, $F^{Sm}_{L_{12,12}}(\phi)$ and $F^{Sm}_{F_{12,12}}(\phi)$. ***Scenario 3* (S3):** in the first two rounds of the game, firms *i* and *j* adopt *tech 1* or *tech 2* (*tech 1(2)*). Firm *i* adopts first (first round) and becomes the leader, firm *j* adopts second (second round) and becomes the follower. Then, at the third and fourth rounds of the game, both firms adopt the remaining technology available *tech 2(1)*, again, one after the other, firm *i* first and firm *j* second[11], and the firms' payoffs are given by $F^{Sq}_{L_{12,12}}(\phi)$ and $F^{Sq}_{F_{12,12}}(\phi)$, respectively for firm *i* and *j*. ***Scenario 4* (S4):** in this scenario, in the first round, firm *i* adopts both

[11] Note that here we have two possibilities: firm *i* adopts the remaining technology first and firm *j* second or the other way round. For simplicity, we assume that the first to adopt the first technology, *tech 1(2)*, is also the first to adopt the second technology, *tech 2(1)*.

technologies simultaneously (*tech 12*) first, and becomes the leader with a payoff given by $F^{Sq}_{L_{12,12}}(\phi)$. Firm *j* adopts then the two technologies sequentially, *tech 1(2)* first and then *tech 2(1)* second and gets $F^{Sq}_{F_{12,12}}(\phi)$ as payoff.

In the next section we derive analytical expressions for the firms' payoffs marked in Figure 1 with an ellipse (S1, S2 and S3). In Figures 2 and 3 below we represent in the time horizon the investment thresholds of the leader and the follower, for the case where the two technologies are adopted sequentially and the case where both technologies are adopted simultaneously, respectively.

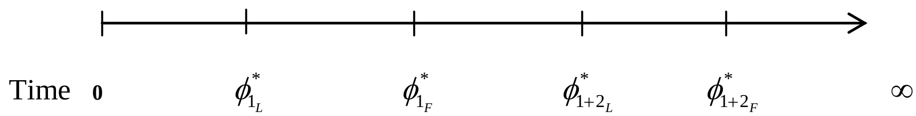


**Figure 2** – Firms' Investment Thresholds when the two Technologies are adopted Sequentially.

Figure 2 represents the investment scenario where both firms adopt, one after the other, the two technologies sequentially; $\phi^*_{1_L}$ represents the leader's investment threshold to adopt *tech 1*, given that none of the technologies have been adopted; $\phi^*_{1_F}$ denotes the follower's investment threshold to adopt *tech 1*, when the leader is operating with *tech 1* and the follower is not yet in the market, $\phi^*_{1+2_L}$ is the leader's investment threshold to adopt *tech 2* given that *tech 1* is in place; and $\phi^*_{1+2_F}$ is the follower's investment threshold to adopt *tech 2* given that it has adopted *tech 1* and the leader is already operating with both *tech 1* and *tech 2*.

Note that in Figure 2 we assume that it is optimal for the follower to adopt *tech 1* before the leader has adopted *tech 2*. For details about the conditions under which this game equilibrium holds see Appendix A.

Figure 3 represents the firms' investment threshold for the scenario where at the beginning of the investment game none of the technologies have been adopted and the two firms, one after the other, adopt the two technologies simultaneously; $\phi^*_{12_L}$ and $\phi^*_{12_F}$ represent, respectively, the leader's and the follower's investment thresholds for this investment scenario.

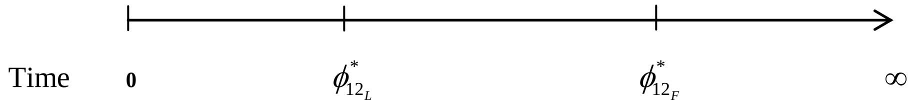


**Figure 3** – Firms' Investment Thresholds when the two Technologies are adopted Simultaneously.

A summary of the firms' investment threshold treated in the paper is given in Table 1.

| Firms' Investment Trigger Values | **The Adoption of *Tech 1* or *Tech 2* alone*** | | **Sequential Adoption**** (***tech 1*/*tech 2***) | **Simultaneous Adoption** (***tech 1* + *tech 2***) |
|---|---|---|---|---|
| **Leader** | $\phi^*_{1_L}$ **Equation (41)** | $\phi^*_{2_L}$ **Equation (41)** | $\phi^*_{1+2_L}$ **Equation (29)** | $\phi^*_{12_L}$ **Equation (45)** |
| **Follower** | $\phi^*_{1_F}$ **Equation (37)** | $\phi^*_{2_F}$ **Equation (37)** | $\phi^*_{1+2_F}$ **Equation (24)** | $\phi^*_{12_F}$ **Equation (43)** |

* The expressions for the firms' threshold to adopt tech 1 and tech 2 are exactly the same, only the subscripts (1, 2) change.
** In the derivation of these expressions we assumed that firms adopt first *tech 1*, but they would not change if we had assumed the other way around, a part from the subscripts (1, 2) which should be exchanged.

**Table 1 -** Investment Thresholds for the Scenarios where Firms adopt the two Technologies, Sequentially and Simultaneously.

### 2.1 The Pre-emption Game

In games of timing the adoption of new technologies, the potential advantage from being the first to adopt may introduce an incentive for preempting the rival, speeding up the first adoption. Fudenberg and Tirole (1985) studied the adoption of a new technology and illustrate the effects of preemption in games of time. We use their concept of preemption to derive the firms' value functions and their investment thresholds.

## 3. The Model

In our model, at the beginning of the investment game there are two new (complementary) technologies available, *tech 1* and *tech 2*, and two idle firms, *i* and *j*, which are considering the adoption of the two technologies, one after the other or both simultaneously depending on which one of these choices is the best. In addition, it is assumed that there is a "first-mover cost savings advantage", the firms' cost savings are a proportion of their revenues, and the evolution of revenues the cost of each technology is uncertain.

The firms' cost savings flow is given by the following expression:

$$X(t)\gamma_k\left[ds_{k_ik_j}\right] \tag{1}$$

where, $X(t)$ is the market revenue flow, $\gamma_k$ represents the proportion of firm's revenues that is expected to be saved through the adoption of technology *k*, with $k=\{0,1,2,12\}$, where 0 means that firm is not yet active and 1, 2 and 12 mean that firm operates with *tech 1* only, with *tech 2* only or with *tech 1* and *tech 2* and the same time, respectively; $ds_{k_ik_j}$ is a deterministic factor that ensures a first-mover revenue advantage, with $i, j=\{L,F\}$, where *L* means "leader" and *F* "follower", and represents the proportion of the market revenues that is held by each firm (*i*, *j*) for each investment scenarios (see inequality 2).

The intuition used here to justify the first-mover "revenue advantage" is similar to that used by Dixit and Pindyck (1994), following Smets (1993). We implicitly assume that firms are symmetric in their ability to operate with the new technologies and that spillover information is not allowed, meaning that the firms' "first-mover revenue advantage" holds forever. Consequently, for the leader, inequality (2) holds:

$$\left(ds_{1_L0_F}=ds_{2_L0_F}=ds_{12_L0_F}\right)>ds_{12_L1_F}>ds_{12_L12_F}>\left(ds_{1_L1_F}=ds_{2_L2_F}\right) \tag{2}$$

The economic interpretation of inequality (2) is the following: for firm *L* (the leader), the best investment scenario is when it adopts both technologies, *tech 1* and *tech 2*, and its opponent, firm *F* (the follower), is inactive ($ds_{12_L0_F}$); its second best scenario is when it adopts first *tech 1* or *tech 2*, $\left[ds_{1_L0_F}=ds_{2_L0_F}\right]$[12]; its third best investment scenario is when it adopts both technologies first and its opponent adopts, later, only *tech 1* ($ds_{12_L1_F}$); its fourth best investment scenario is when both firms adopt both technologies but the leader does so earlier ($ds_{12_L12_F}$); its fifth best investment scenario is when both firms adopt one technology, *tech 1* or *tech 2*, but the leader does so earlier

[12] In inequality (2) we assume, through $ds_{1_L0_F}=ds_{2_L0_F}$ and $ds_{2_L2_F}=ds_{1_L1_F}$, that the leader's first-mover advantage is the same regardless of the technology it chooses. However, our framework allows the use of different assumptions on this regard.

$\left[ds_{2_L 2_F} = ds_{1_L 1_F}\right]$; and finally, its worst investment scenario is when it is inactive ( $ds_{0_L 0_F}$ )[13]. For More details on the competition factors and their influence on the game equilibrium see Appendix A.

We assume that the variable market revenues, $X(t)$, follows a geometric Brownian motion given by the following equation:

$$dX = \mu_X Xdt + \sigma_X Xdz_X \tag{3}$$

where, $\mu_X$ is the trend rate of growth of market revenues, $\sigma_X$ is the volatility of the market revenues and $dz_X$ is the increment of a standard Wiener process.

We consider that *tech 1* alone provides a net cost savings, $S_1$, that is a fraction, $\gamma_1$, of the firm's market revenues, $X\left[ds_{k_i k_j}\right]$:

$$S_1 = \gamma_1 X\left[ds_{k_i k_j}\right] \tag{4}$$

Since the firms' cost savings are proportional to revenues and revenues follow a gBm process, so firms' cost savings also follows a gBm process. The equation for that process is:

$$dS_1 = \mu_{S_1} S_1 dt + \sigma_{S_1} S_1 dz_{S_1} \tag{5}$$

where, $\mu_{S_1}$ is the trend rate of growth of the cost savings due to the adoption of *tech 1*, $\sigma_{S_1}$ is the volatility of the cost savings when *tech 1* is adopted and $dz_{S_1}$ is the increment of a standard Wiener process.

Similarly, the use of *tech 2* alone provides a cost savings equal to:

$$S_2 = \gamma_2 X\left[ds_{k_i k_j}\right] \tag{6}$$

with,

[13] Note that by assumption the leader is the firm that adopts first, therefore, the scenario in which the follower is active and the leader is inactive is not possible.

$$dS_2 = \mu_{S_2} S_2 dt + \sigma_{S_2} S_2 dz_{S_2} \tag{7}$$

where, $\mu_{S_2}$ is the trend rate of growth of the cost savings due to the adoption of *tech 2*, $\sigma_{S_2}$ is the volatility of the cost savings when *tech 2* is adopted and $dz_{S_2}$ is the increment of a standard Wiener process.

The simultaneous use of both technologies yields cost savings equal to:

$$S_{12} = \gamma_{12} X \left[ ds_{k_i k_j} \right] \tag{8}$$

with, $S_{12}$ also following a gBm process, given by Equation (9):

$$dS_{12} = \mu_{S_{12}} S_{12} dt + \sigma_{S_{12}} S_{12} dz_{S_{12}} \tag{9}$$

where, $\mu_{S_{12}}$ is the trend rate of growth of the cost savings due to the adoption of both *tech 1* and *tech 2*, $\sigma_{S_{12}}$ is the volatility of the cost savings when both technologies are adopted and $dz_{S_{12}}$ is the increment of a standard Wiener process.

The technological complementary between the two technologies is ensured by the following inequality:

$$\gamma_{12} > \gamma_1 + \gamma_2 \tag{10}$$

Furthermore, we assume that the costs of adopting *tech 1* and *tech 2*, respectively, $I_1$ and $I_2$, follow gBm processes as well, given by:

$$dI_1 = \mu_{I_1} I_1 dt + \sigma_{I_1} I_1 dz_{I_1} \tag{11}$$

and

$$dI_2 = \mu_{I_2} I_2 dt + \sigma_{I_2} I_2 dz_{I_2} \tag{12}$$

where, $\mu_{I_1}$ and $\mu_{I_2}$ are the trend rates of growth of the cost of *tech 1* and *tech 2*, respectively; $\sigma_{I_1}$ and $\sigma_{I_2}$ are the volatility of the cost of *tech 1* and *tech 2*, respectively; and $dz_{I_1}$ and $dz_{I_2}$ are the increments of the standard Wiener processes for the costs *tech 1* and *tech 2*, respectively.

## 3.1 Technology 1 is in place

### 3.1.1 The Follower's Value Function

In this session we derive the follower's option value to adopt *tech 2* assuming that *tech 1* is in place, $f_{12}(X, I_2)$. Once we have $f_{12}(X, I_2)$, we will derive the expression for the total value $F_{12}(X, I_2) = V_1 + f_{12}(X, I_2)$, where $V_1$ is the follower's expected value from operating with *tech 1* forever, and given by expression (13):

$$V_1 = \frac{\gamma_1 X \left[ ds_{k_i k_j} \right]}{r - \mu_X} \tag{13}$$

Setting the returns on the option equal to the expected capital gain on the option and using Ito's lemma, we obtain this partial differential equation (PDE) for the value function of an active follower (i.e., a follower which is operating with *tech 1*) in the region in which it waits to adopt *tech 2*:

$$\frac{1}{2}\sigma_X{}^2 X^2 \frac{\partial^2 F_{12}}{\partial X^2} + \frac{1}{2}\sigma_{I_2}^2 I_2{}^2 \frac{\partial^2 F_{12}}{\partial I_2{}^2} + X I_2 \sigma_X \sigma_{I_2} \rho_{XI_2} \frac{\partial^2 F_{12}}{\partial X \partial I_2} + \mu_X X \frac{\partial F_{12}}{\partial X} + \mu_{I_2} I_2 \frac{\partial F_{12}}{\partial I_2} + \gamma_1 X \left( ds_{1_F k_L} \right) = r F_{12} \tag{14}$$

where, $\rho_{XI_2}$ is the correlation coefficient between the market revenues, *X*, and the cost of *tech 2*, $I_2$ and *r* is the riskless interest rate.

Equation (14) must be subjected to two boundary conditions. The first is the *"value matching"* condition:

(i) There is a value of $F_{12}(X, I_2)$ at which the follower will invest and at that point in time the follower's value equals the present value of the cash flows minus the investment costs ($I^*_{2_F}$):

$$F_{12}(X,I_2)=\frac{(\gamma_{12}-\gamma_1)X^*\left[ds_{k_ik_j}\right]}{r-\mu_X-\mu_{I_2}}-I^*_{2_F} \tag{15}$$

where, $(\gamma_{12}-\gamma_1)X^*\left[ds_{k_ik_j}\right]$ represents the follower's cost savings at the time the follower adopts *tech 2*; $X^*\left[ds_{k_ik_j}\right]$ is the follower's revenues at the time of adoption; $(\gamma_{12}-\gamma_1)$ is the proportion of the follower's revenues that is expected to be saved due to the adoption of *tech 2* on the assumption that *tech 1* is already in place; $X^*$ and $I^*_{2_F}$ are, respectively, the market revenue and the cost of *tech 2* at the follower's adoption time.

The second boundary condition comes from the *"smooth pasting"* conditions, for the value of both the idle and the active follower:

(ii) The first derivative, with respect to both stochastic variables, $X(t)$ and $I_2(t)$, at the point where the value functions equal the present value of the cash flows, $(X/I_2)^*$. Therefore, it holds that:

$$\frac{\partial F_{12}(X,I_2)^*}{\partial X^*}=\frac{(\gamma_{12}-\gamma_1)\left[ds_{k_ik_j}\right]}{r-\mu_X-\mu_{I_2}} \tag{16}$$

$$\frac{\partial F_{12}(X,I_2)^*}{\partial I^*_{2_F}}=-1 \tag{17}$$

In the present case, the natural homogeneity of the investment problem, i.e., $F_{12}(X,I_2)=I_2 f_{12}(X/I_2)$, where $f_{12}$ is the variable to be determined, allows us to reduce it to one dimension. Using the following change in the variables $\phi_2=X/I_2$ and substituting this relation in the PDE (14) yields[14]:

$$\frac{1}{2}\sigma_{m_2}(\phi_2)^2\frac{\partial^2 f_{12}(\phi_2)}{\partial\phi_2{}^2}+\left(\mu_X-\mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial\phi_2}-(r-\mu_{I_2})f_{12}(\phi_2)+\gamma_1 X\left(ds_{1_L1_F}\right)=0 \tag{18}$$

where, $\sigma^2_{m_2}=\sigma^2_X+\sigma^2_{I_2}-2\rho_{XI_2}\sigma_X\sigma_{I_2}$.

[14] For a detailed computation of Equation (18) see Appendix B.

Equation (18) is a homogeneous second-order linear ordinary differential equation (ODE) whose general solution has the form:

$$f_{1+2}(\phi_2) = A_{1+2}(\phi_2)^{\beta_1} + B_{1+2}(\phi_2)^{\beta_2} \tag{19}$$

where, $\beta_{1(2)}$ is the characteristic quadratic function of the homogeneous part of equation (18), given by:

$$\frac{1}{2}(\sigma_{m_2})^2 \beta_1(\beta_1 - 1) + (\mu_X - \mu_{I_2})\beta_1 - (r - \mu_{I_2}) = 0 \tag{20}$$

Solving the equation above for $\beta_1$ leads to:

$$\beta_1 = \frac{1}{2} - \frac{\mu_X - \mu_{I_2}}{\sigma_{m_2}^2} + \sqrt{\left(\frac{(\mu_X - \mu_{I_2})}{\sigma_{m_2}^2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_{I_2})}{\sigma_{m_2}^2}} \tag{21}$$

Note that as the ratio of market revenues to cost of *tech 2,* $\phi_2$ , approaches 0, the value of the option to adopt *tech 2* becomes worthless; therefore, in Equation (19) $B_{1+2} = 0$.

Rewriting the boundary conditions we obtain following "value-matching" condition:

$$f_{1+2}(\phi^*_{1+2_F}) = \frac{(\gamma_{12} - \gamma_1)\left[ds_{k_i k_j}\right]\phi^*_{1+2_F}}{r - \mu_X - \mu_{I_2}} - 1 \tag{22}$$

where, $\phi_2 = \phi^*_{1+2_F}$ is the follower's investment threshold to adopt *tech 2* given that *tech 1* is already in place, and "smooth-past" condition:

$$\frac{\partial f_{1+2}(\phi^*_{1+2_F})}{\partial \phi^*_{1+2_F}} = \frac{(\gamma_{12} - \gamma_1)\left[ds_{12_L 12_F}\right]}{r - \mu_X - \mu_{I_2}} \tag{23}$$

Solving together equations (19), (22) and (23) we get the following value for $\phi^*_{1+2_F}$ and the constant $A_{12}$:

$$\phi^*_{1+2_F} = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu_X - \mu_{I_2})}{\left(ds_{12_F 12_L}\right)(\gamma_{12} - \gamma_1)} \tag{24}$$

$$A_{1+2} = \frac{\left(\phi^*_{1+2_F}\right)^{-\beta_1}}{\beta_1 - 1} \frac{(\gamma_{12} - \gamma_1)\left(ds_{12_F 12_L}\right)}{r - \mu_X - \mu_{I_2}} \tag{25}$$

where, $\varphi^*_{1+2_F}$ is the follower's threshold for adopting *tech 2* if *tech 1* has been adopted.

Finally, using equations (19), (24) and (25) we derive the follower's value function:

$$F^{SQ}_{F_{12,12}}(\phi_2) = \begin{cases} \dfrac{\gamma_1 X\left(ds_{1_F 12_L}\right)}{r - \mu_X} - I^*_{1_F} + A_{12}\left(\dfrac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1} I_2 & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma_{12} X\left(ds_{12_F 12_L}\right)}{r - \mu_X} - I^*_{2_F} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \tag{26}$$

Equation (25) tells us that, before $\phi^*_{1+2_F}$, the follower's value when it adopts the two technologies sequentially is given by the value of operating with *tech 1* forever, $\frac{\gamma_1\left(ds_{1_F 12_L}\right)X}{r - \mu_X} - I^*_{1_F}$, plus its option to adopt *tech 2*, $\frac{I_2}{\beta_1 - 1}\left(\frac{\phi_2}{\phi^*_{1+2_F}}\right)^{\beta_1}$, as soon as $\phi^*_{1+2_F}$ is reached, the follower's value is equal to the net present value of the cost savings obtained by the follower if it operates with both technologies from $\phi^*_{1+2_F}$ until infinity, $\frac{\gamma_{12} X\left(ds_{12_F 12_L}\right)}{r - \mu_X} - I^*_{2_F}$.

### 3.1.2 The Leader's Value Function

Assuming that both firms are operating with *tech 1* and that the follower will adopt *tech 2* at $\varphi^*_{1+2_F}$ (derived above), the leader's value function is described by the following expression[15]:

$$E\left[\int_{T_{2_L}}^{T_{2_F}} \gamma_{12} X_\tau \left(ds_{12_L 1_F}\right) e^{-r\tau} d\tau - I^*_{2_L} + \int_{T_{2_F}}^{\infty} \gamma_{12} X_\tau \left(ds_{12_L 12_F}\right) e^{-r\tau} d\tau\right] \tag{27}$$

[15] Note that it is assumed that *tech 1* is in place.

where, the first integral represents the leader's cost savings in the period where it operates with the two technologies and the follower operates with *tech 1*; the second integral represents the leader's cost savings for the period where both firms are operating with the two technologies, *tech 1* and *tech 2*; $I^*_{2_L}$ is the cost of *tech 2* at the leader's adoption time.

Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader's value function:

$$F^{SQ}_{L_{12,12}}(\phi_2) = \begin{cases} \dfrac{\gamma_{12} X \left( ds_{12_L 1_F} \right)}{r - \mu_X} - I^*_{1_L} - I^*_{2_L} + \dfrac{\beta_1}{\beta_1 - 1} \left( \dfrac{\phi_2}{\phi^*_{1+2_F}} \right)^{\beta_1} \gamma_{12} \left( ds_{12_L 12_F} - ds_{12_L 1_F} \right) I_2 & \phi_2 < \phi^*_{1+2_F} \\ \dfrac{\gamma_{12} X \left( ds_{12_L 12_F} \right)}{r - \mu_X} - I^*_{1_L} - I^*_{2_L} & \phi_2 \geq \phi^*_{1+2_F} \end{cases} \tag{28}$$

Expression $\dfrac{\gamma_{12} X \left( ds_{12_L 1_F} \right)}{r - \mu_X} - I^*_{1_L} - I^*_{2_L}$ corresponds to the leader's total payoff if it operates alone with the two technologies forever; $\left( \dfrac{\phi_2}{\phi^*_{1+2_F}} \right)^{\beta_1} \gamma_{12} \left( ds_{12_L 12_F} - ds_{12_L 1_F} \right) I_2 \dfrac{\beta_1}{\beta_1 - 1}$ is negative, since $\left( ds_{12_L 12_F} - ds_{12_L 1_F} \right) < 0$ (see inequality (2)), and corresponds to the correction factor that incorporates the fact that in the future if $\phi^*_{1+2_F}$ is reached the follower will adopt *tech 2* and the leader's profits will be reduced.

We do not get a closed-form solution for the leader's trigger value. However, a numerical solution can be determined applying numerical methods to the equation (29), where $\phi^*_{1+2_L}$ replaces $\phi_2$ and is the unknown variable. Equation (29) is derived by equalizing the value functions of the leader and the follower, for $\phi_2 < \phi^*_{1+2_F}$.

$$\frac{\gamma X \left( ds_{12_L 1_F} \right)}{r - \mu_X} - I^*_{1_L} - I^*_{2_L} + \frac{\beta_1}{\beta_1 - 1} \left( \frac{\phi_2}{\phi^*_{1+2_F}} \right)^{\beta_1} \gamma_{12} \left( ds_{12_L 12_F} - ds_{12_L 1_F} \right) I_2 - \frac{\gamma_1 X \left( ds_{1_F 12_L} \right)}{r - \mu_X} + I^*_{1_F} - A_{12} \left( \frac{\phi_2}{\phi^*_{1+2_F}} \right)^{\beta_1} I_2 \tag{29}$$

### 3.2 None of the Technologies have been adopted

Now that we have the value of the implicit option on *tech 2* if *tech 1* has been adopted, we can analyze the first-stage decision to adopt *tech 1*. Similarly as we have done for the scenario where we assume that *tech 1* is in place, here we derive the firms' value functions and investment trigger values for the scenario where neither of the technologies has been adopted.

### 3.2.1 The Follower's Value Function

Let $F(X, I_1, I_2)$ be the value of the option to adopt either one or both technologies. Setting the return on the option $rF$ equal to the expected capital gain on the option and using Ito's lemma, we obtain this differential equation for the region in which the firm waits to invest:

$$0 = \frac{1}{2}\left( \sigma_X^2 X^2 \frac{\partial^2 F}{\partial X^2} + \sigma_{I_1}^2 I_1^2 \frac{\partial^2 F}{\partial I_1^2} + \sigma_{I_2}^2 I_2^2 \frac{\partial^2 F}{\partial I_2^2} + 2\rho_{XI_1}\sigma_X \sigma_{I_1} XI_1 \frac{\partial^2 F}{\partial C \partial I_1} + 2\rho_{XI_2}\sigma_X \sigma_{I_2} XI_2 \frac{\partial^2 F}{\partial X \partial I_2} + \ldots \right.$$
$$\left. \ldots + 2\rho_{I_1 I_2}\sigma_{I_1}\sigma_{I_2} I_1 I_2 \frac{\partial^2 F}{\partial I_1 \partial I_2} \right) + \mu_X X \frac{\partial F}{\partial X} + \mu_{I_1} I_1 \frac{\partial F}{\partial I_1} + \mu_{I_2} I_2 \frac{\partial F}{\partial I_2} - rF \tag{30}$$

where, $\rho_{XI_1}$ and $\rho_{XI_2}$ are the correlation coefficients between the market revenues and the cost of *tech 1* and the market revenues and the cost of *tech 2*, respectively, and $\rho_{I_1 I_2}$ is the correlation coefficient between the cost of *tech 1* and the cost of *tech 2*.

In the region where the firm is waiting to adopt, this value can be separated into the value of the option to acquire *tech 1* plus the value of the option to acquire *tech 2* as well. Assuming first-order homogeneity, i.e., $F(X, I_1, I_2) = I_1 f_1(X / I_1) + I_2 f_{12}(X / I_2)$, the relevant partial derivatives yield:

$$0 = \left( \frac{1}{2}\sigma_{m_1}\phi_1 \frac{\partial^2 f_1(\phi_1)}{(\partial\phi_1)^2} + (\mu_X - \mu_{I_1})\frac{\partial f_1(\phi_1)}{\partial(\phi_1)} - (r - \mu_{I_1})\phi_1 f_1 \right) + \ldots$$
$$\ldots + \left( \frac{1}{2}\sigma_{m_2}\phi_2 \frac{\partial^2 f_{12}(\phi_2)}{(\partial\phi_2)^2} + (\mu_X - \mu_{I_2})\frac{\partial f_{12}(\phi_2)}{\partial(\phi_2)} - (r - \mu_{I_2})\phi_2 f_{12} \right) \tag{31}$$

where, $\sigma_{m_1} = \sigma_X^2 - 2\rho_{XI_1}\sigma_X \sigma_{I_1} + \sigma_{I_1}^2$ and $\sigma_{m_2} = \sigma_X^2 - 2\rho_{XI_2}\sigma_X \sigma_{I_2} + \sigma_{I_2}^2$.

In the region where the current value of the ratio "market revenues" over "cost of *tech 2*" is lower than the threshold to adopt *tech 2* if *tech 1* is already in place, i.e., in the region where $\phi_2 \le \phi^*_{1+2_F}$

(see Equation 24), the second bracketed expression is equal to zero, leaving this second-order linear differential equation[16] equal to:

$$0=\left(\frac{1}{2}\sigma_{m_1}(\phi_1)\frac{\partial^2 f_1(\phi_1)}{(\partial\phi_1)^2}+(\mu_X-\mu_{I_1})\frac{\partial f_1(\phi_1)}{\partial(\phi_1)}-(r-\mu_{I_1})\phi_1 f_1\right) \tag{32}$$

Therefore, the economically meaningful solution is:

$$f_1(\phi_1)=A_1(\phi_1)^{\beta_1}+B_1(\phi_1)^{\beta_2} \tag{33}$$

where, $\beta_{1(2)}$ is the characteristic quadratic function of the homogeneous part of equation (31), given by:

$$\frac{1}{2}(\sigma_{m_1})^2\beta_1(\beta_1-1)+(\mu_X-\mu_{I_1})\beta_1-(r-\mu_{I_1})=0 \tag{34}$$

Solving the equation above for $\beta_1$ leads to:

$$\beta_1=\frac{1}{2}-\frac{\mu_X-\mu_{I_1}}{\sigma_{m_1}^2}+\sqrt{\left(\frac{(\mu_X-\mu_{I_1})}{\sigma_{m_1}^2}-\frac{1}{2}\right)^2+\frac{2(r-\mu_{I_1})}{\sigma_{m_1}^2}} \tag{35}$$

As the ratio revenues over the cost of tech 1, $\phi_1$, approaches 0, the value of the option does too; therefore, $B_1=0$. Using the "*value matching*" and the "*smooth pasting*" conditions at the threshold ratio, $\phi_{1_F}^*$, we obtain:

$$A_1=\frac{\phi_{1_F}^{*\,-\beta_1}}{\beta_1-1}\frac{\left(ds_{1_F1_L}\right)\gamma_1}{r-\mu_X-\mu_{I_1}} \tag{36}$$

$$\phi_{1_F}^*=\frac{\beta_1}{\beta_1-1}\frac{(r-\mu_X-\mu_{I_1})}{\left(ds_{1_F1_L}\right)\gamma_1} \tag{37}$$

[16] The assumption underlying the derivation is that given that the investment threshold to adopt tech 2 if tech 1 is in place is $\phi_{1+2_F}^*$ and that due to the effect of complementarity between tech 1 and tech 2 this threshold is always lower than the threshold to adopt *tech 2* alone, $\phi_{2_F}^*$, so before $\phi_{1+2_F}^*$ it is not optimal to adopt *tech 2* alone, i.e., the option to adopt tech 2 is worthless.

$$F_{F_{1,1}}(\phi_1) = \begin{cases} \dfrac{I_1}{\beta_1 - 1}\left(\dfrac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1} \dfrac{\left(ds_{1_F 1_L}\right)\gamma_1}{r - \mu_X - \mu_{I_1}} & \phi_1 < \phi_{1_F}^* \\ \dfrac{\gamma_1 X\left(ds_{1_F 1_L}\right)}{r - \mu_X} - I_{1_F}^* & \phi_1 \geq \phi_{1_F}^* \end{cases} \tag{38}$$

Since we did not differentiate the two technologies, the expressions for the case of *tech 2* are exactly the same as those derived above for the case of the adoption of *tech 1*. The only difference is the subscript used in the notation for the complementarity parameters and the competition factors, where the subscript "2" replaces "1".

Notice that $\phi_{1_F}^*$ is the follower's threshold for adopting *tech 1* by itself and $\phi_{1+2_F}^*$ is the follower's threshold to adopt *tech 2* given that *tech 1* is in place. From these expressions we conclude that when the two technologies are complements, the degree of complementarity does not affect the decision to adopt either technology by itself, but does reduce the threshold for adopting the other if one technology is adopted. This result was obtained by Smith (2005) for a context where competition is absent.

### 3.2.2 The Leader's Value Function

Focusing again on the adoption of *tech 1*, the leader's expected value is given by:

$$E\left[\int_{T_{1_L}}^{T_{1_F}} \gamma_1 X_\tau \left(ds_{1_L 0_F}\right) e^{-r\tau} d\tau - I_{1_L}^* + \int_{T_{1_F}}^{\infty} \gamma_1 X_\tau \left(ds_{1_L 1_F}\right) e^{-r\tau} d\tau\right] \tag{39}$$

The first integral represents the leader's payoff when alone in the market; the second integral represents the leader's payoff when operating with the follower, both with *tech 1*; $I_{1_L}^*$, is the cost of *tech 1* at the leader's adoption time.

Applying the methodology used in Dixit and Pindyck (1994), pp. 309-315, we get the following expression for the leader's value function:

$$F_{L_{1,1}}(\phi_1)=\begin{cases}\dfrac{\gamma_1 X\left(ds_{1_L0_F}\right)}{r-\mu_X}-I_{1_L}^* + \dfrac{\beta_1}{\beta-1}\left(\dfrac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1}\left(ds_{1_L1_F}-ds_{1_L0_F}\right)I_1 & \phi_1<\phi_{1_F}^* \\ \dfrac{\gamma_1 X\left(ds_{1_L1_F}\right)}{r-\mu_X}-I_{1_L}^* & \phi_1\geq\phi_{1_F}^*\end{cases} \tag{40}$$

Again, we do not get a closed-form solution for the leader's trigger value. However, a numerical solution can be determined applying numerical methods to the equation (41), where $\phi_{1_L}^*$ is the unknown variable. Equation (41) is obtained by equalizing the value functions of the leader and the follower, for $\phi_1 < \phi_{1_F}^*$.

$$\frac{\gamma_1 X\left(ds_{1_L0_F}\right)}{r-\mu_X}+\frac{\beta_1}{\beta_1-1}\left(\frac{\phi}{\phi_{1_F}^*}\right)^{\beta_1}\left(ds_{1_L1_F}-ds_{1_L0_F}\right)I_1-I_{1_L}^*-\frac{I_1}{\beta_1-1}\left(\frac{\phi_1}{\phi_{1_F}^*}\right)^{\beta_1}\frac{\left(ds_{1_F1_L}-ds_{0_F1_L}\right)\gamma_1}{r-\mu_X-\mu_{I_1}} \tag{41}$$

The procedure used to get this equation was the same as that used for Equation (28).

### 3.3 Simultaneous Adoption

Following similar procedures we get the expressions for the firms' value functions and investment trigger values for the case where, for some technical/economic reasons, the two technologies have to be adopted simultaneously. For simplicity of notation we use $I_1+I_2=I_{12}$.

#### 3.3.1 The Follower's Value Function

$$F_{F_{12,12}}^{SM}(\phi_{12})=\begin{cases}\dfrac{I_{12}}{\beta_1-1}\left(\dfrac{\phi_{12}}{\phi_{12_F}^*}\right)^{\beta_1} & \phi_{12}<\phi_{12_F}^* \\ \dfrac{\gamma_{12} X\left(ds_{12_F12_L}\right)}{r-\mu_X}-I_{12_F}^* & \phi_{12}\geq\phi_{12_F}^*\end{cases} \tag{42}$$

Investment threshold value:

$$\phi_{12_F}^* = \frac{\beta_1}{\beta_1-1}\frac{(r-\mu_X-\mu_{I_{12}})}{\left(ds_{12_F12_L}\right)\gamma_{12}} \tag{43}$$

### 3.3.2 The Leader's Value Function

$$F^{SM}_{L_{12,12}}(\phi_{12}) = \begin{cases} \dfrac{\gamma_{12} X\left(ds_{12_L 0_F}\right)}{r-\mu_X} - I^*_{12_L} + \dfrac{\beta_1}{\beta_1 - 1}\left(\dfrac{\phi_{12}}{\phi^*_{12_F}}\right)^{\beta_1}\left(ds_{12_L 12_F} - ds_{12_L 0_F}\right) I_{12} & \phi_{12} < \phi^*_{12_F} \\ \dfrac{\gamma_{12} X\left(ds_{12_L 12_F}\right)}{r-\mu_X} - I^*_{12_L} & \phi_{12} \geq \phi^*_{12_F} \end{cases} \tag{44}$$

Investment threshold value:

$$\frac{\gamma_{12} X\left(ds_{12_L 0_F}\right)}{r-\mu_X} - I^*_{12_L} + \frac{\beta_1}{\beta_1 - 1}\left(\frac{\phi_{12}}{\phi^*_{12_F}}\right)^{\beta_1}\left(ds_{12_L 12_F} - ds_{12_L 0_F}\right) I_{12} - \frac{I_{12}}{\beta_1 - 1}\left(\frac{\phi_{12}}{\phi^*_{12_F}}\right)^{\beta_1} = 0 \tag{45}$$

## 4. Results and Sensitivity Analysis

In this section we analyse the sensitivity of our real options model to changes in some of its most important parameters. To illustrate our results we use the following inputs: $X = 60$, $I_1 = 7.0$, $I_2 = 7.0$, $I^*_{1_L} = 6.0$, $I^*_{1_F} = 5.0$, $I^*_{2_L} = 6.0$, $I^*_{2_F} = 5.0$, $\sigma_X = 0.4$, $\sigma_{I_1} = \sigma_{I_2} = 0.20$, $\mu_X = 0.05$, $\mu_{I_1} = -0.05$, $\mu_{I_2} = -0.10$, $r = 0.09$, $\rho_{XI_1} = \rho_{XI_2} = \rho_{XI_{12}} = 0$, $\gamma_1 = 0.10$, $\gamma_2 = 0.10$, $\gamma_{12} = 0.30$. The competition factors used are: $ds_{1_L 0_F} = ds_{2_L 0_F} = ds_{12_L 0_F} = 1.0$, $ds_{1_L 1_F} = ds_{2_L 2_F} = ds_{12_L 1_F} = 0.60$, $ds_{1_F 1_L} = ds_{2_F 2_L} = ds_{1_F 12_L} = 0.40$, $ds_{12_L 12_F} = 0.55$, $ds_{12_F 12_L} = 0.45$.

According to our inputs the unique asymmetry between the two technologies concerns their cost growth rates. More specifically, we assume that the cost of tech 1 is expected to fall at an annual rate of 10 percent ($\mu_{I_1} = -0.1$), and the cost of tech 2 is expected to fall at an annual rate of 5 percent ($\mu_{I_2} = -0.05$). In figures 6 and 7 below we show the investment thresholds of an idle leader and follower, respectively, for the scenarios where *tech 1* is adopted alone, *tech 2* is adopted alone and *tech 1* and *tech 2* are adopted at the same time. In table 2, $\Phi_1(t)$, $\Phi_2(t)$ and $\Phi_{12}(t)$, represent the current value of the ratios "revenues/cost of tech 1", "revenues/cost of tech 2", and "revenues/the sum of the costs of tech 1 and tech 2", respectively; $\Phi^*_{1L}$, $\Phi^*_{2L}$ and $\Phi^*_{12L}$ expresses the leader's investment thresholds to adopt *tech 1* alone, *tech 2* alone and *tech 1 and tech 2* at the same time, respectively; and $\Phi^*_{1F}$, $\Phi^*_{2F}$ and $\Phi^*_{12F}$ expresses the follower's investment thresholds to adopt *tech 1* alone, *tech 2* alone and *tech 1* and *tech 2* at the same time, respectively. Given that in our framework the value of investment depends on two stochastic underlying variables, X and $I_k$, so the investment threshold for each firm and investment scenario is defined by a straight line plotted in the $(X,I_k)$ space. Each threshold line represents the firm's investment threshold for a particular strategy. Each threshold line has a different slope, the higher the slope, the later is the adoption.

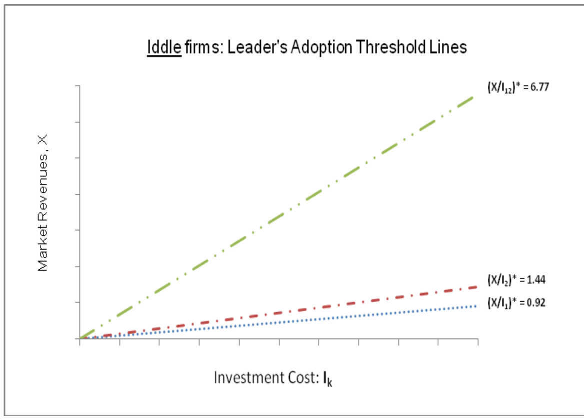


**Figure 6**

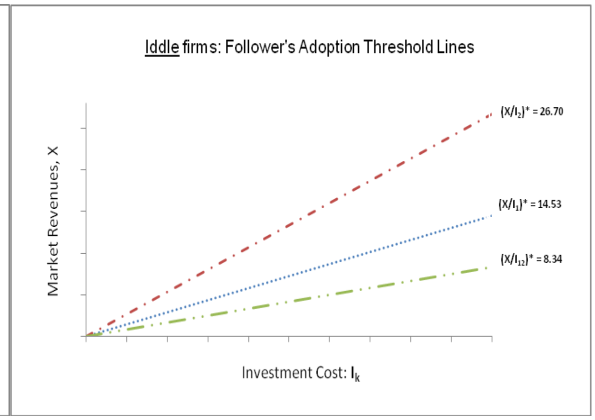


**Figure 7**

| **Current Values** | | | **Follower's Thresholds** | | | | | **Leader's Threshold** | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Idle Firm | | | Tech 1 In place | Tech 2 In Place | Idle Firm | | | Tech 1 In place | Tech 2 In Place |
| $\Phi_1(t)$ | $\Phi_2(t)$ | $\Phi_{12}(t)$ | $\Phi^*_{1,F}$ | $\Phi^*_{2,F}$ | $\Phi^*_{12,F}$ | $\Phi^*_{1+2,F}$ | $\Phi^*_{2+1,F}$ | $\Phi^*_{1,L}$ | $\Phi^*_{2,L}$ | $\Phi^*_{12,L}$ | $\Phi^*_{1+2,L}$ | $\Phi^*_{2+1,L}$ |
| **8.57** | **8.57** | **4.29** | 14.53 | 26.70 | 8.34 | 19.81 | 6.46 | 0.92 | 1.44 | 6.77 | 0.25 | 0.11 |
| ***Investment decision:*** | | | ***wait*** | ***wait*** | ***wait*** | ***wait*** | ***invest*** | ***invest*** | ***invest*** | ***wait*** | ***invest*** | ***invest*** |

**Table 2**

In our model, in expression (1), $\gamma_k$ represents the proportion of firms' revenues that is expected to be saved through the adoption of technology k. Inequality (10) ensures that the functions of the two technologies are complements. More specifically, in our inputs we use $\gamma_1 = 0.10$, $\gamma_2 = 0.10$ and $\gamma_{12} = 0.30$. Therefore, $\gamma_1 + \gamma_2 = 0.20$, but $\gamma_{12} = 0.30$, hence the complementarity between tech 1 and tech 2, i.e., firms have a 10 percent "extra revenues incentive", $\gamma_{12} - (\gamma_1 + \gamma_2) = 0.10$, to operate with both technologies at the same time. For investment scenarios where uncertainty is absent and adjustment costs are neglected, traditional analysis have shown that synchronous adoption is always optimal. Our results show, however, that that is not always the case if uncertainty is taken into account.

Our results reported in figures 6 and 7 show that the optimal investment behavior for the follower is to adopt tech 1 and tech 2 at the same time as soon as $\Phi^*_{12,F}$=8.34 is crossed and the optimal investment behaviour for the leader is to adopt the two technologies sequentially, first, tech 1 as soon as $\Phi^*_{1,L}$=0.92 is reached and then tech 2 as soon as $\Phi^*_{2,L}$=1.44 is crossed. In addition we can see that for all investment scenarios, the leader adopts before the follower (as expected) and for both firms, in case sequential adoption is optimal, the technology whose price is decreasing more slowly (tech 1) is adopted first and the technology whose price is decreasing more rapidly (tech 2), is adopted after tech 1 is in place, i.e., the thresholds to adopt tech 1 alone is lower than the threshold to adopt tech 2 alone for scenarios available. These are the results for the scenarios where at the beginning of the investment game both firms are idle. If so, firms have the options to adopt tech 1 alone, tech 2 alone and tech 1 and tech 2 at the same time. These are independent options. Firms do exercise the one which fits with, or belongs to, the optimal investment strategy for a particular investment circumstance.

In figures 8 and 9 we present our results for the case where at the beginning of the investment game both firms are active, operating with tech 1 or tech 2. If at the beginning of the investment game both firms are active with tech 1, then they have the option to adopt tech 2, if at the beginning of the investment game both firms are active with tech 2, then they have the option to adopt tech 1[17]. We determined the leader and follower investment thresholds for each one of these cases. Below are our results.

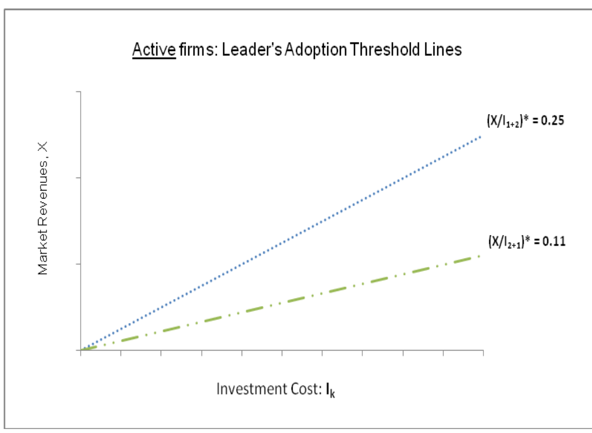


**Figure 8**

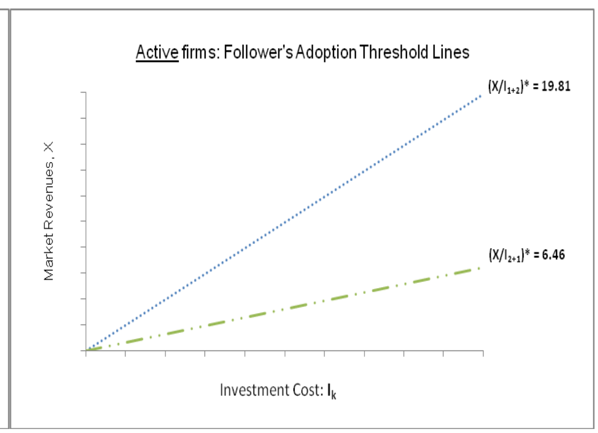


**Figure 9**

From the results above we can see that when tech 1 is in place alone, the leader should adopt tech 2 as soon as the current value of the ratio "revenues/cost of tech 2", $\Phi_2(t)$, reaches $\Phi^*_{1+2,L}$=0.25 and

[17] Note that once one of the technologies is in place the option to adopt both technologies at the same time is eliminated.

the follower should adopt tech 2 as soon as the current value of the ratio “revenues/cost of tech 2”, $\Phi_2(t)$, reaches $\Phi^*_{1+2,F}$=19.81. When *tech 2* is in place alone, the leader should adopt tech 1 as soon as the current value of the ratio “revenues/cost of tech 1”, $\Phi_1(t)$, reaches $\Phi^*_{2+1,L}$=0.11 and the follower should adopt tech 1 as soon as the current value of the ratio “revenues/cost of tech 1”, $\Phi_1(t)$, reaches $\Phi^*_{2+1,F}$=6.46.

Looking at the expressions for the leader and the follower investment thresholds for the scenario where at the beginning of the investment game both firms are active with one of the technologies (see Equations (29) and (24) respectively), we can see that the degree of complementarity between tech 1 and tech 2 does not affect the decision to adopt either technologies alone, but reduces the threshold for adopting one if the other is adopted. Regarding the effect of the volatility of the underlying variables (revenues and cost of technology k) on firms’ investment thresholds, usual comments apply, i.e., the higher the volatility of the underlying variables, the later is the adoption of the respective technology(ies) for both firms.

In Figures 10 and 11 below, show our results concerning the effect of the expected rate of decline in the cost of the tech 1 and tech 2 on the leader and the follower optimal investment thresholds, for each of the investment scenarios available. Given that for most industries, due to the innovation pace, the price of the technologies falls over time, in our simulations we use negative growth rates, more specifically in the range (-20% to -2%).

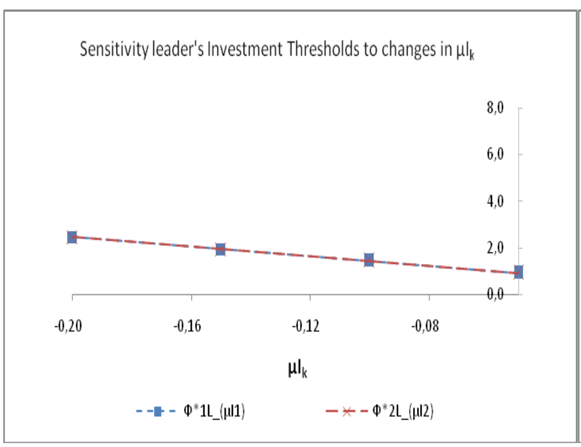


**Figure 10**

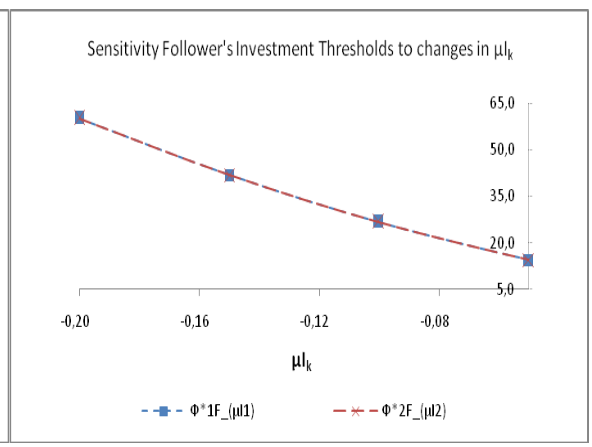


**Figure 11**

In Figures 10 and 11 there are two overlapped lines which represent the sensitivity of the leader and the follower investment thresholds, respectively, to adopt tech 1 alone and tech 2 alone to changes in the expected rate of decline of their respective costs. The firms’ investment threshold to adopt tech 1 alone and to adopt tech 2 alone match because in this study we did not differentiate the two technologies[18]. Or results show that, for the leader and the follower[19], the higher the expected rate of decline in the price of the technology the higher are their investment thresholds, i.e., the later the adoption of the respective technology. This result was expected given that the faster the decline in the price of a technology the higher is the incentive to delay the adoption.

We also study the impact of the difference between the cost growth rates of the two technologies on the leader and the follower investment thresholds. In Figure 12 we present our results.

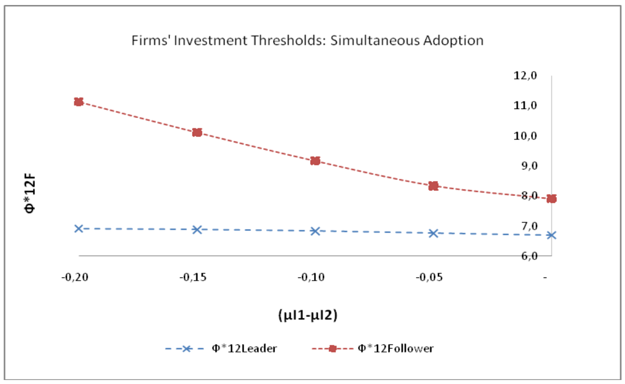


**Figure 12**

The results show that both the leader and the follower investment threshold are sensitive to changes in the difference between the cost growth rates of the two technologies. The higher the difference between the expected rates of decline in the price of the two technologies, the later (or less likely) is the adoption of both technologies at the same time, although this effect is more evident for the follower than for the leader. The differences between the degree of sensitivity of the leader's investment threshold and the degree of sensitivity of follower's investment threshold to changes in the amplitude of the difference between the rates of decline in the prices of the two technologies, is due to the preemption effect. Note that in our inputs we assume that the leader while alone in the market gets 100 percent of the market revenues ($ds_{12_L 0_F} = 1.0$) and that as soon as the follower

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[18] Note that in our inputs (see page 22) we assume that the tech 1 and tech 2 are symmetric except in their expected rate of cost decline (we use $\mu_{I_1} = -0.05$ for tech 1 and $\mu_{I_2} = -0.10$ for tech 2). Therefore, when we set $\mu_{I_1} = \mu_{I_2}$, being the rest of the model parameters equal, the investment thresholds to adopt each of the technology alone should match.

[19] Although from figures 8 and 9 we can see that the follower's investment threshold is slightly more sensitive to changes in the expected rate of decline in the price of the technology than is the leader's investment threshold. This happens because for low values of complementarity between the two technologies, the first-mover advantage effect predominates for the leader and does not have any affect for the follower given that, as soon as the leader invests, the follower is a monopoly like.

adopts the tech 1 and tech 2 its market share revenues is reduced to 55 percent ($ds_{12_L 12_F} = 0.55$), and the follower gets the remaining 45 percent ($ds_{12_F 12_L} = 0.45$). As *X*, the market revenues, is assumed to be 60 million, so the first-mover advantage represents 10 percent of 60 million (6 million). Our results show that this 6 million "extra revenues" is much more important than its potential gains from delaying the adoption of the two technologies. The follower is by assumption the firm which invests after the leader. Therefore, it will not benefit from the first-mover advantage and consequently, is more sensitive to the potential gains from a decline in the price of the two technologies.

Finally, in figures 12 and 13 we present our results for the sensitivity analysis of the impact of the degree of complementarity between tech 1 and tech 2 on the leader and the follower investment thresholds to adopt the two technologies at the same time.

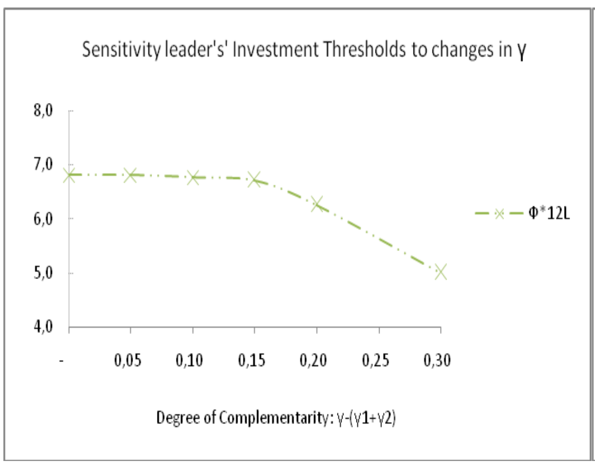


**Figure 12**

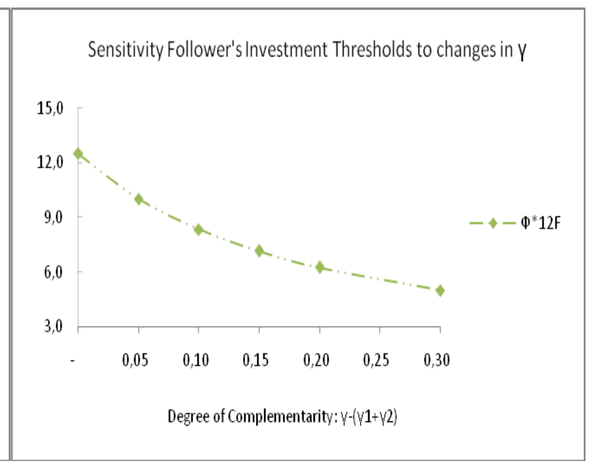


**Figure 13**

In our model we define $\gamma_1$, $\gamma_2$ and $\gamma_{12}$ as the proportion fo the total market revenues which can be saved due to the adoption of tech 1 alone, tech 2 alone and tech 1 and tech 2 at the same time, respectively. Therefore, the higher each of these parameters the higher is the firms' expected revenues when active and, therefore, the lower should be their investment threshold, i.e., the ealier the adoption of the respective technology(ies). Note that the option to adopt tech 1 is independent of the option to adopt tech 2. Thus, as we can see from expression (37), $\gamma_2$ does not affect the firms' investment threshold to adopt tech 1 and $\gamma_1$ does not affect the firms' investment threshold to adopt tech 2[20]. The option to adopt tech 1 and tech 2 at the same time is also independent of the options to adopt tech 1 alone and tech 2 alone. The proportion of the market revenues that can be save when tech 1 and tech 2 are adopted at the same time, $\gamma_{12}$, affects only the firms' investment threshold to adopt the two technologies at the same time, but not the optimal time to adopt any of the technologies alone.

Being $\gamma_1$ and $\gamma_2$ constant, the higher the $\gamma_{12}$ the higher is the complementarity between tech 1 and tech 2 and, therefore, the earlier the adoption of both technologies at the same time for leader and the follower. This intuition is confirmed by the results presented in Figures 12 and 13. The differences of sensitivity between the leader and the follower are justified by the first-mover advantage, which affects the leader's behaviour and does not affect the follower's.

## 5. Conclusions and Further Research

Our results show that, in a *ceteris paribus* analysis, the higher the degree of complementarity between the two technologies, the earlier is the adoption of both technologies at the same time and the more advantageous is such decision compared with the adoption of each technology alone. In addition, we found that when the degree of complementarity between the two technologies is low and the rate of decline in the price of the two technologies differs substantially, it might be optimal for both firms to adopt the two technologies at different times, first the technology whose price is decreasing at a lower rate and then the technology whose price is decreasing more rapidly. This is an important result in the since that it contradicts the conventional wisdom which says that "when a production process requires two extremely complementary inputs, a firm should upgrade (or replace) them simultaneously". Furthermore, we show that the degree of complementarity between two technologies does not affect the decision to adopt either technologies alone, but reduces the threshold for adopting one technology if the other is adopted.

We study the effect of the complementarity between two technologies on the leader and the follower investment decisions, considering uncertainty and competition. Our investment game setting is built under the assumption that there is a first-mover (*pre-emption* game) advantage. However, an interesting extension for this research would be to derive a similar investment model, but for an economic context where a second-mover advantage (*war of attrition* game) holds. In addition, in our framework we assume that there are two firms (the leader and the follower) and two

[20] Note that we did not differentiate the two technologies and, therefore, the expression for the case of the investment threshold to adopt *tech 2* is exactly the same as that derived for the case of the adoption of *tech 1* (equation 37). The only difference is the subscript used in the notation, where "2" replaces "1".

technologies which can be adopted at the same time or at different times. Given that it is quite common to find projects that have more than two inputs whose functions are complement, an interesting research would be to extend this model to investments with more than two complementary inputs. The extension of this model to oligopoly markets, although technically challenging, would be an interesting and useful research as well.

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**Appendix A**

1. **Differential Equations: Homogeneity of degree one**

In this paper we use similarity methods to get a closed-form solution for the differential equation (14). Examining the value "matching conditions" we can easily prove that homogeneity exists. Therefore, taking the "value matching" condition below (see equation 15, p. 13) we have:

$$F_{12}(X,I_2)=\frac{(\gamma-\gamma_1)\left[ds_{k_ik_j}\right]X^*}{r-\mu_X-\mu_{I_2}}-I_{2_F}^* \tag{A1}$$

Therefore, if the option value is $F_{12}(X,I_2)$ and the value after exercising the option is $\frac{(\gamma-\gamma_1)\left[ds_{k_ik_j}\right]X^*}{r-\mu_X-\mu_{I_2}}-I_{2_F}^*$, with both $X$ (market revenues) and $I_2$ (investment) stochastic, then if $F_{12}(X,I_2)=\frac{(\gamma-\gamma_1)\left[ds_{k_ik_j}\right]X^*}{r-\mu_X-\mu_{I_2}}-I_{2_F}^*$ holds, doubling $X$ and $I_2$ doubles $F_{12}(X,I_2)$, so there is homogeneity of degree one. If the "value matching" relationship exhibits homogeneity of degree one, then it exists and the two variables $(X,I_2)$ can be replaced by, in this case, the ratio $X/I_2=\phi_2$.

2. **The Deterministic Factors**

In our framework the leader's first-mover market advantage, altogether with the assumption about the technological complementarity, is ensured by inequality (2) (see p. 10) where each of the deterministic factors represents the leader's market share for each investment scenario, given as a proportion of the total market.

$$\left(ds_{1_L0_F}=ds_{2_L0_F}=ds_{12_L0_F}\right)>ds_{12_L1_F}>ds_{12_L12_F}>\left(ds_{1_L1_F}=ds_{2_L2_F}\right) \tag{A2}$$

For instance, for a market value of 10 million dollars if we set $ds_{12_L12_F}=0.6$ this means that when both firms are operating with tech 1 and tech 2 at the same time, the leader gets 60 percent of the market revenues (6 million) and the follower the remaining 40 percent (4 million), given that for a duopoly market the sum of the market share of the leader with the market share of the follower is equal to 100 percent (i.e., $ds_{12_L12_F}+ds_{12_F12_L}=1.0$, hence if $ds_{12_L12_F}=0.6$, then $ds_{12_F12_L}=0.4$). In addition inequality (2) means that when the leader operates with both technologies at the same time its market share is higher if the follower is active operating with one technology alone than if the follower is active operating with both technologies at the same time as well (hence $ds_{12_L 1_F} > ds_{12_L 12_F}$). The reason for this is that when the follower operates with one technology alone it does not benefit from the effect of the complementarity between the two technologies. When the leader is alone in the market it gets 100 percent of the market revenues, regardless of with what technology(ies) it is operating with, tech 1 alone, tech 2 alone or tech 1 and tech 2 at the same time. Hence, $ds_{1_L 0_F} = ds_{2_L 0_F} = ds_{12_L 0_F} = 1.0$. Inequality (2) also shows that the best scenario for the leader is when it is alone in the market, for obvious reasons.

Our investment model is set as a "zero-sum pre-emption game" with two firms competing for a percentage of the total market revenues. For each firm and investment scenario we deterministically assign a revenues market share. The relative market revenues advantage assigned to each strategy is guided by inequality (2). We derive the firms' payoffs and their respective investment thresholds for some specific investment game scenarios (those marked in figure 1 with an ellipse), combining the real options theory with the Fudenberg and Tirole (1985, pp. 386-389) game-theoretic arguments. Backed by inequality (2), we then can compare the value functions of the leader and the follower (firms' payoffs) for the investment strategies available and, consequently, the leader and the follower optimal decision at each of game tree nodes (see figure 1, p. 7).

### 3. The Firms' Payoffs

In our investment game we have two firms and two technologies which can be adopted at the same time or at different times. Consequently, the number of investment scenarios grows substantially when compared with investment games with two firms but with only one technology or with the case where there are two technologies involved in the investment decision but they cannot be adopted at the same time. However, at each node of the game-tree, the use of the information underlying inequality (2), as explained above, simplifies our work regarding the determination of the firms' optimal strategy. For instance, using expression (1) as the general expression for the firms' value functions:

$$X(t)\gamma_k \left[ ds_{k_i k_j} \right] \tag{1}$$

where, $X(t)$ is the market revenue flow, $\gamma_k$ represents the proportion of firm's revenues that is expected to be saved through the adoption of technology *k*, with $k = \{0, 1, 2, 12\}$, where 0 means that firm is not yet active and 1, 2 and 12 mean that firm operates with *tech 1* only, with *tech 2* only or with *tech 1* and *tech 2* and the same time, respectively; $ds_{k_i k_j}$ is the proportion of the market share, deterministically assigned to each firm and investment scenario, which ensures a first-mover revenue advantage, with $i, j = \{L, F\}$, where *L* means "leader" and *F* "follower". Taking *i* as the leader and *j* as the follower, $ds_{12_i 1_j} > ds_{12_i 12_j}$ becomes $ds_{12_L 1_F} > ds_{12_L 12_F}$, hence the leader's revenues market share is higher when it operates with tech 1 and tech 2 and the follower operates with tech 1 only ($ds_{12_L 1_F}$) than when the leader operates with tech 1 and tech 2 and the follower as well ($ds_{12_L 12_F}$). This logic/procedure is applied at each node of the investment game-tree to determine the firms' optimal investment strategy and the game equilibrium.

**Appendix B**

**1. Derivation of the Ordinary Differential Equation (18)**

Equation (14) is written as:

$$\frac{1}{2}\frac{\partial^2 F_{12}}{\partial X^2}\sigma_X{}^2 X^2+\frac{1}{2}\frac{\partial^2 F_{12}}{\partial I_2{}^2}\sigma_{I_2}^2 I_2{}^2+\frac{\partial^2 F_{12}}{\partial X\,\partial I_2}XI_2\sigma_X\sigma_{I_2}\rho_{XI_2}+\frac{\partial F_{12}}{\partial X}\mu_X X+\frac{\partial F_{12}}{\partial I_2}\mu_{I_2}I_2-rF_{12}=0$$

In order to reduce the homogeneity of degree two in the underlying variables to homogeneity of degree one, similarity methods can be used. Let $\phi_2=\frac{X}{I_2}$, so:

$$F(X,I_2)=F(\phi_2)$$

$$\frac{\partial F(X,I_2)}{\partial I_2}=\frac{\partial F(\phi_2)}{\partial \phi_2}X$$

$$\frac{\partial F(X,I_2)}{\partial X}=\frac{\partial F(\phi_2)}{\partial \phi_2}I_2$$

$$\frac{\partial^2 F(X,I_2)}{\partial I^2}=\frac{\partial^2 F(\phi_2)}{(\partial I)^2}X^2$$

$$\frac{\partial^2 F(X,I_2)}{\partial X^2}=\frac{\partial^2 F(\phi_2)}{(\partial \phi_2)^2}(I_2)^2$$

$$\frac{\partial^2 F(X,I_2)}{\partial X\ \partial I_2}=\frac{\partial^2 F(\phi_2)}{(\partial \phi_2)^2}XI_2+\frac{\partial F(\phi_2)}{\partial \phi_2}$$

Substituting back to Equation (14) we obtain Equation (18):

$$\frac{1}{2}\sigma_{m_2}(\phi_2)^2\frac{\partial^2 f_{12}(\phi_2)}{\partial \phi_2{}^2}+\left(\mu_X-\mu_{I_2}\right)(\phi_2)\frac{\partial f_{12}(\phi_2)}{\partial \phi_2}+\gamma_1 X\left(ds_{1_L 1_F}\right)-(r-\mu_{I_2})f_{12}(\phi_2)=0$$