Section 2

Transmission Lines and Cables

Introduction

In a power system, power is transferred from one busbar to another via a network of transmission lines, usually in the form of three-phase overhead lines or, in densely populated urban areas, cables. In this section we will look at how transmission lines are modelled, look at the factors that affect the physical parameters of the line and examine the relationship between line voltages and the flow of power and reactive power along the line.

Learning Outcomes

On completion of this section you will be able to:

- Model the behaviour of three-phase transmission overhead lines and cables using a simple per phase circuit representation.
- Carry out circuit analysis for short and medium length transmission lines.
- Describe the relationship between line voltages and line power and reactive power flows.
- Calculate real and reactive power flows between two busbars with known voltages
- Calculate busbar voltages from a knowledge of power and reactive power line flows
- Calculate line power losses

Time

You will need between 2 and 3 hours for this section.

Resources

Calculator, pen and paper.

2.1 Circuit Representation of Transmission Lines

Fig. 2.1 shows a single-phase line consisting of a conductor suspended above the ground. The line is characterised by its resistance *R*, its series self-inductance *L*, its shunt capacitance to earth *C* and its shunt leakage conductance *G*, which represents the leakage current to earth.

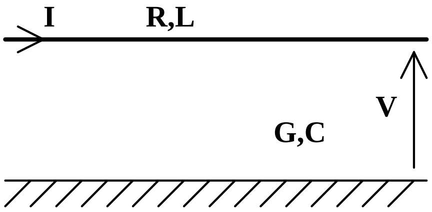


Fig. 2.1 Single-phase line

A three-phase line includes mutual inductance and capacitance effects, but each of its' three conductors can be modelled by an equivalent single-phase line with modified parameters.

2.1.1 Distributed parameter model

All these parameters are of course distributed along the entire length of the line (Fig. 2.2) and are usually expressed in Ohms per unit length (Ω/km), Henry per unit length (H/km), Farad per unit length (F/km) and Siemens per unit length (S/km).

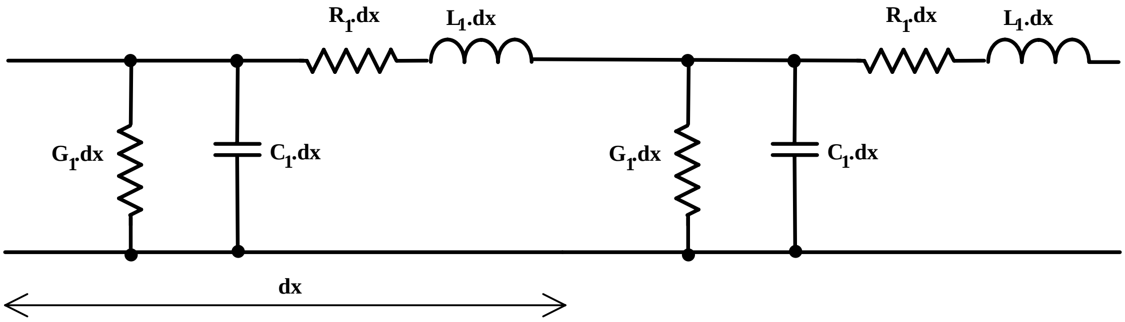


Fig. 2.2 Distributed parameter model.

2.1.2 Nominal line models

It is usual practice to perform circuit calculations using *lumped* circuit elements obtained simply by multiplying the distributed parameters by the length of the line. A line is usually represented by either the T network or the π network shown in Fig. 2.3.

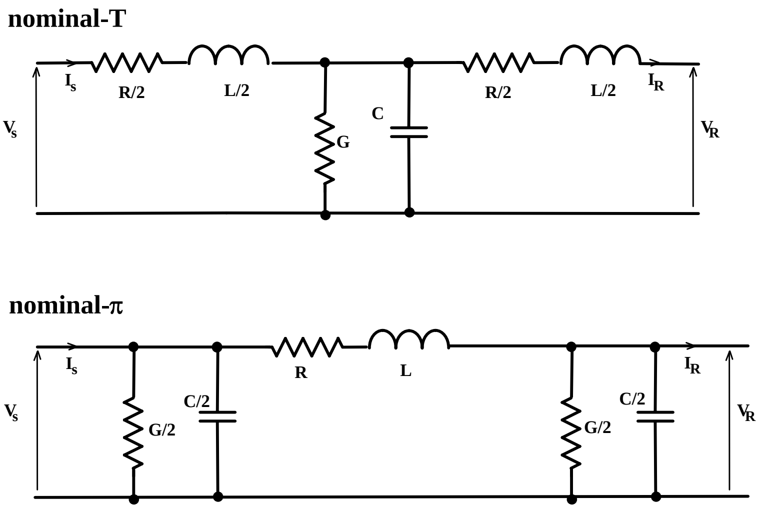


Fig. 2.3 Nominal line models

2.1.3 Short-line model

For fully-loaded lines less than 100km long, the current flow in the shunt elements is less than 1% of the full-load current. In this case, the shunt elements may be neglected giving the short line model shown in Fig. 2.4.

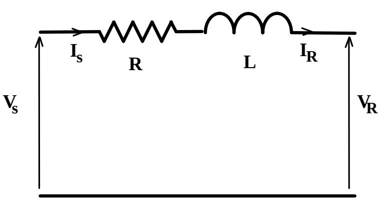


Fig. 2.4 Short line model

Normally the series reactance of the line is much bigger than the resistance and it may sometimes be possible, especially for short urban lines, to disregard the resistance giving the simplified line model shown in Fig. 2.5.

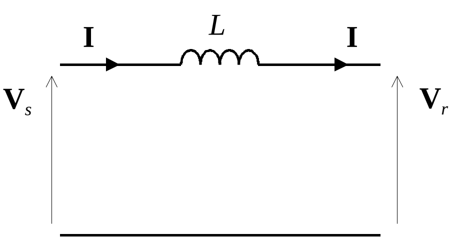
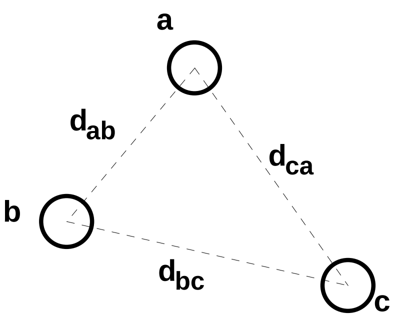


Fig. 2.5 Simplified short line representation

2.2 Relationships between Line Parameters and Physical Layout

2.2.1 Line inductance

Fig. 2.6 shows the relationship between physical layout of a three phase overhead line and its' series line inductance.



$$L = \frac{\mu_0}{2\pi}.ln\left\{\frac{D}{R}\right\} \quad H.m^{-1}$$

where:

μ_0 = *permeability of space* $= 4\pi \times 10^{-7} \quad H.m^{-1}$

R = *effective conductor radius*

D = *geometric mean distance* $= \sqrt[3]{d_{ab} \times d_{bc} \times d_{ca}}$

Fig. 2.6 Line inductance

To simplify operation of the power system it is desirable to have low values of inductance, i.e. small *D* and large *R*.

Decreasing *D* reduces the insulation between lines and is therefore limited by the working voltage.

Increasing *R* causes an increase in conductor weight and therefore cost.

One method employed to increase the effective radius is the use of bundle conductors, as shown in Fig. 2.7.

Cross-section of line for one-phase in the UK transmission system:

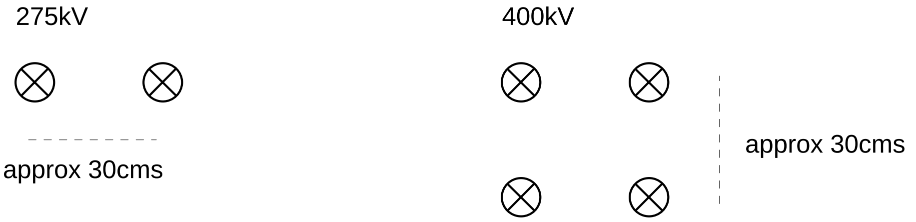


Fig. 2.7 Bundle conductors

In comparison to a single solid conductor, bundle conductors:

- increase effective radius and therefore reduce inductance
- reduce skin effect
- have a larger surface area and therefore better cooling
- are easier to handle during construction

2.2.2 Line resistance

The resistance of a line varies between 0.5Ω/km for an 11kV distribution line and 0.015Ω/km for a 400kV overhead line or a 33kV underground cable.

Resistance includes skin effect, which causes an increase in resistance of about 5% (in comparison to dc) in a 2.5cm diameter copper conductor operating at 50 Hz.

2.2.3 Line capacitance

The capacitance per unit length of the line shown in Fig. 2.6 above is given by the equation:

$$C = \frac{2\pi\varepsilon_0}{\ln(D/R)}$$

for a given working voltage and frequency, dv/dt is fixed, so to minimise charging current, i ($i = C\, dv/dt$), the line capacitance should be as small as possible. A low value of C implies large D and small R conflicting with the requirements for small L.

Typical values for capacitive reactance (1/ωC) are 200kΩ/km for a transmission line and 4kΩ/km for an underground cable. The capacitive charging current in an underground cable is thus much higher than in an overhead transmission line.

2.2.4 Line conductance

Line shunt conductance G models losses due to corona (discharge through air) and leakage currents across insulator surfaces. Typical losses on a 400KV line are 600 W/km in fine weather and 90 kW/km in snow or fog.

2.3 Underground Cables vs. Overhead Lines

Cables are 15-20 times more expensive than overhead lines, because:

- insulation cost (overhead lines uses air insulation, which is free)
- the maximum operating temperature for a cable is typically 70 or 90°C, so more copper must be used to reduce losses and give a reasonable operating temperature
- installation cost: trench, continuous path across the ground

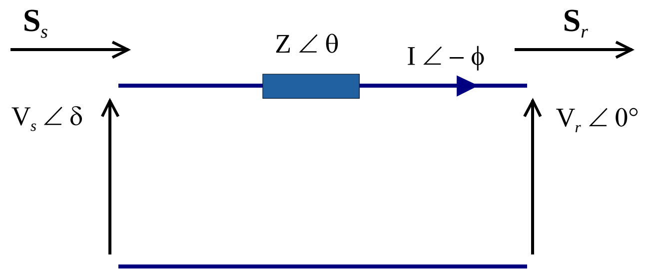
Plus, the large capacitive charging current limits useful lengths of cables to 15-20 km. For longer lengths of cable (e.g. under the sea) dc transmission is employed.

Fig. 2.8 Undersea cable dc link arrangement

2.4 Line Power and Reactive Power Flows

2.4.1 Line Power Flow

The receiving end voltage $\mathbf{V}_r$ is shown as our reference phasor in Fig. 2.9. ϕ is the angle by which the current **I** lags the receiving-end voltage $\mathbf{V}_r$, θ is the phase angle of the line impedance given by arctan $(\omega L / R)$ and δ is the angle by which the sending-end voltage $\mathbf{V}_s$ leads $\mathbf{V}_r$. A phasor diagram representation of the above quantities is also shown.



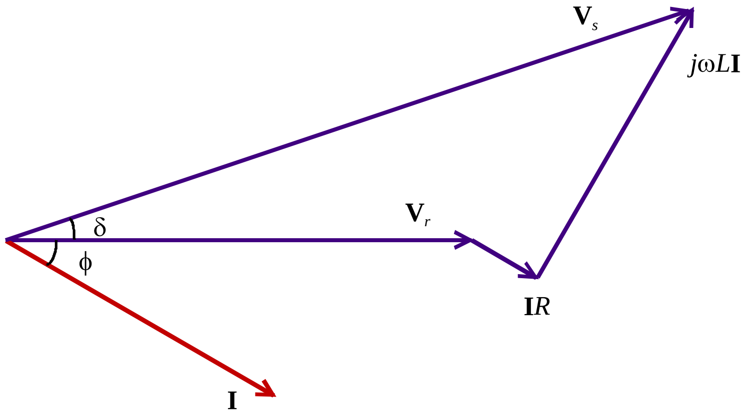


Fig. 2.9 Phasor diagram for short transmission line

We can calculate the complex power at each end from the equations

$$\mathbf{S}_S = P_s + j\,Q_s = \mathbf{V}_S\,\bar{\mathbf{I}} \tag{2.1}$$

$$\mathbf{S}_r = P_r + Q_r = \mathbf{V}_r\,\bar{\mathbf{I}} \tag{2.2}$$

where $\bar{\mathbf{I}} = \left(\frac{\bar{\mathbf{V}}_s - \bar{\mathbf{V}}_r}{\bar{\mathbf{Z}}}\right)$ (2.3)

Substituting (2.3) in (2.1) and (2.2)

$$\mathbf{S}_S = \mathbf{V}_S\left(\frac{\bar{\mathbf{V}}_S - \bar{\mathbf{V}}_r}{\bar{\mathbf{Z}}}\right) = V_S \angle\delta\left(\frac{(V_S \angle -\delta) - V_r}{Z \angle -\theta}\right) = \frac{V_S\,V_S}{Z}\angle\theta - \frac{V_S\,V_r}{Z}\angle(\theta+\delta) \tag{2.4}$$

$$\mathbf{S}_r = \mathbf{V}_r\left(\frac{\bar{\mathbf{V}}_S - \bar{\mathbf{V}}_r}{\bar{\mathbf{Z}}}\right) = V_r\left(\frac{(V_S \angle -\delta) - V_r}{Z \angle -\theta}\right) = \frac{V_r\,V_S}{Z}\angle(\theta-\delta) - \frac{V_r\,V_r}{Z}\angle\theta \tag{2.5}$$

Thus,

$$P_s = \frac{V_s\, V_s}{Z} \cos\theta - \frac{V_s\, V_r}{Z} \cos(\theta + \delta) \text{ (Watt per phase)} \tag{2.6}$$

$$P_r = \frac{V_r\, V_s}{Z} \cos(\theta - \delta) - \frac{V_r\, V_r}{Z} \cos\theta \text{ (Watt per phase)} \tag{2.7}$$

Equations (2.6) and (2.7) may be simplified further if we make the practical assumption that the line resistance R is negligible compared with its reactance ωL. In other words, if we assume that $R = 0$, $Z = \omega L = X_L$ and $\theta = 90°$. Because we are neglecting the line resistance, the real line powers at each end are equal and given by:

$$P_s = P_r = \frac{V_s\, V_r}{X_L} \sin\delta \text{ (Watt per phase)} \tag{2.8}$$

Equation 2.8 is very important in understanding the limits of power transmission capability of power lines. The power output to the receiving-end is maximum when $\sin\delta = 1$, i.e. when $\delta = 90°$.

Note that the busbar voltages at both line ends V_s and V_r are practically constant and can only change within very tight limits. Hence, the only way in which we can vary the real power transmitted through the line is to vary the angle δ, usually referred to as the *load angle* or sometimes the *power angle*.

Equation (2.8) shows that as the load power increases, δ increases to a maximum value of 90°. The maximum line power $P_{\max}$ is then given by:

$$P_{\max} = \frac{V_s\, V_r}{X_L} \text{ (Watt per phase)} \tag{2.9}$$

Any further attempt to increase line power will cause the transmission system to collapse, i.e. the system will step out of synchronism. We have in fact arrived at the *static stability limit* of the line.

If the flow of power is reversed, meaning that power transfer is from right to left in Fig. 2.8, δ will be negative (i.e. $\mathbf{V}_s$ lagging $\mathbf{V}_r$)

Interestingly, the above analysis tells us that in an ac power system, the voltage magnitudes at either end of a transmission line do not determine the direction of power transfer along the line. The voltage phase difference does.

The magnitude and direction of the flow of real power on a line depends on the phase angle between the sending end voltage and the receiving end voltage. Power flows from the end with the leading voltage to the end with the lagging voltage. The magnitude of power flowing down the line increases with an increasing phase difference.

2.4.2 Line Reactive Power Flow

From (2.4) and (2.5), we may write:

$$Q_s = \frac{V_s\ V_s}{Z}\sin\theta - \frac{V_s\ V_r}{Z}\sin(\theta+\delta) \text{ (VAr per phase)} \tag{2.10}$$

$$Q_r = \frac{V_r\ V_s}{Z}\sin(\theta-\delta) - \frac{V_r\ V_r}{Z}\sin\theta \text{ (VAr per phase)} \tag{2.11}$$

Ignoring line resistance *R*, equation (5.17) may be written as:

$$Q_r = \frac{V_r\ V_s}{X_L}\cos\delta - \frac{V_r\ V_r}{X_L} \text{ (VAr per phase)} \tag{2.12}$$

Under normal operating conditions, $\cos\delta$ is pretty close to unity and (2.12) may be simplified further to:

$$V_s - V_r \approx \frac{X_L\ Q_r}{V_r} \tag{2.13}$$

We can express this by saying that the amount of reactive VArs consumed at the receiving end is roughly proportional to the voltage difference in voltage magnitudes at either end of the line, *the greater the voltage difference the stronger the flow of reactive power*. Also, *the flow of reactive power is in the direction of the lowest voltage*. In other words, if there is a difference between the magnitudes of the voltages at the sending and receiving ends of the line, lagging reactive power will tend to flow towards the end with the lower voltage.

Another way of looking at this is to say that if there is a deficiency of reactive power at a point in the network (i.e. a lagging reactive power load), this will have to be supplied from the connecting lines and the voltage at that point will fall. Conversely, if there is a surplus of reactive power generated at a point (i.e. a leading power factor load), then the voltage will rise. This means that if we can manage to supply the reactive power requirement of the load locally so that the line reactive power is zero, then there will be no voltage drop between the two ends of the line, which is great.

The magnitude and direction of reactive power flow on a line depends on the difference in magnitude between the sending end voltage and the receiving end voltage. Reactive power flows from the end with the higher voltage to the end with the lower voltage. The magnitude of reactive power increases with an increasing voltage difference.

2.4.3 Line Power Loss

Transmission lines will of course have a certain series resistance R and consequently a power loss P_{loss} given by

$P_{loss} = R\ |\mathbf{I}|^2$ (Watts per phase)

We will now derive an approximate formula for P_{loss} in terms of average line quantities to show how the line power loss varies with line power and reactive power flows.

If we define the average values of line voltage, current, power and reactive power as measured say, at the middle of the line, by the following equations

$P_{av} = (P_s + P_r)/2$, $Q_{av} = (Q_s + Q_r)/2$, $\mathbf{V}_{av} = (\mathbf{V}_s + \mathbf{V}_r)/2$ and $\mathbf{I}_{av} = (\mathbf{I}_s + \mathbf{I}_r)/2$

We can then write the approximate formula

$$P_{av} + j\,Q_{av} \approx \mathbf{V}_{av}\ \bar{\mathbf{I}}_{av}$$

Hence

$$\bar{\mathbf{I}}_{av} \approx \frac{P_{av} + j\,Q_{av}}{\mathbf{V}_{av}}\text{, and } \mathbf{I}_{av} \approx \frac{P_{av} - j\,Q_{av}}{\bar{\mathbf{V}}_{av}}$$

Multiplying the above two equations we obtain

$$\mathbf{I}_{av} \times \bar{\mathbf{I}}_{av} = |\mathbf{I}_{av}|^2 \approx \frac{P_{av} - j\,Q_{av}}{\bar{\mathbf{V}}_{av}} \times \frac{P_{av} + j\,Q_{av}}{\mathbf{V}_{av}} \approx \frac{P_{av}^2 + Q_{av}^2}{V_{av}^2}$$

By substituting for $|\mathbf{I}_{av}|^2$ we obtain the following approximate equation for P_{loss}

$$P_{loss} \approx \frac{R\left(P_{av}^2 + Q_{av}^2\right)}{V_{av}^2} \text{ (Watts per phase)} \qquad (2.14)$$

Equation (2.14) tells us that real and reactive line powers make an equal contribution to the real power loss. We should therefore try to minimise the line reactive power flow if we wish to reduce the power loss. This is another reason why reactive power is often generated at or near the busbar where it is needed, often by installing shunt capacitors for this purpose.

The equation also tells us that the real power loss varies inversely with the square of the line voltage. This is a second very good reason for transmitting power at high voltages.

Exercise 2.1

How much power can be transmitted over a 30 mile long, 33 kV line which has a total reactance of 6.3 Ω per phase and a total resistance of 0.2 Ω per phase?

Turn to the end of the book for suggested answers to the exercise

Exercise 2.2

Have a look at Equations 2.9 and 2.14. Summarise the relationship between line voltage and maximum transmittable power and line voltage and power loss. What practical implications does this have for transmission voltages?

Turn to the end of the book for suggested answers to the exercise

2.5 Transmission Line Calculations

Worked Example 2.1

A three-phase 132 kV, 100 miles long transmission line has a resistance of 0.1 Ω per mile per phase, phase inductance of 1.5 mH per mile per phase, and phase capacitance of 9 nF per mile per phase. Calculate the sending end power if the receiving end voltage is 132 kV and the total receiving end power is (90 + *j* 45), MW and MVArs.

The line is short enough to use lumped circuit parameters and to ignore the effects of shunt leakage, but not short enough to ignore the effects of leakage capacitance. In other words, it is a medium length line and we will use the π network representation of Figure 3.3.

Our **first task** is to compute the lumped circuit parameters.

$R = 0.1 \times 100 = 10\ \Omega$ per phase

$L = 1.5 \times 10^{-3} \times 100 = 0.15$ H ⁄ phase,

$\Rightarrow X_L = 2\pi \times 50 \times 0.15 = 47.1\ \Omega$ ⁄ phase

$C = 9 \times 10^{-9} \times 100 = 0.9 \times 10^{-6}$ F ⁄ phase,

$\Rightarrow Y = j\, 2\pi \times 50 \times 0.9 \times 10^{-6} / 2 = j\, 0.14 \times 10^{-3}\ \Omega^{-1}$ ⁄ phase.

where Y is the admittance of half the lumped C at the end of the line.

We know the receiving-end power and voltage, so we can calculate the receiving-end current $\mathbf{I}_r$.

The per phase complex power at the receiving-end $\mathbf{S}_r$ is given by

$$\mathbf{S}_r = \mathbf{V}_r\, \bar{\mathbf{I}}_r = \frac{90}{3} + j\,\frac{45}{3} = 30 + j\,15 \text{ (MW and MVArs per phase)}$$

Where $\bar{\mathbf{I}}_r$ is the complex conjugate of $\mathbf{I}_r$

As usual, we choose the voltage at the receiving end as our reference phasor (i.e. $\mathbf{V}r = Vr \angle 0°$), so that

$$\bar{\mathbf{I}}_r = \frac{30\times10^6 + j\,15\times10^6}{132\times10^3 / \sqrt{3}} = 393.6 + j\,196.8 \text{ A}$$

Thus

$$\mathbf{I}_r = 393.6 - j\,196.8 \text{ A} = 440 \angle -26.6° \text{ A}$$

Next, we need to calculate the sending-end voltage $\mathbf{V}_s$. To do so we need to compute the voltage drop across the series impedance

$\mathbf{Z} = R + j\,\omega L$.

The shunt current $\mathbf{I}_{shr}$ is given by

$\mathbf{I}_{shr} = \mathbf{V}_r\, \mathbf{Y} = (132 \times 10^3 / \sqrt{3})\,(j\, 0.14 \times 10^{-3}) = j\, 10.7$ A.

The series current $\mathbf{I}_{ser}$ is given by

$\mathbf{I}_{ser} = \mathbf{I}_{shr} + \mathbf{I}_r = 393.6 - j\ 186.1$ A $= 435 \angle -25.3°$ A.

The voltage drop across **Z** is then given by

$\Delta\mathbf{V} = \mathbf{Z}\ \mathbf{I}_{ser}$

$= (10 + j\ 47.1)\ (393.6 - j\ 186.1) = (12.7\times10^3 + j\ 16.7\times10^3)$

$= 20.9\times10^3 \angle 52.7°$ Volts.

The sending-end voltage $\mathbf{V}_s$ is given by

$\mathbf{V}_s = \mathbf{V}_r + \Delta\mathbf{V} = (132 \times 10^3 / \sqrt{3}) + (12.7\times10^3 + j\ 16.7\times10^3)$

$= (88.9\times10^3 + j\ 16.7\times10^3) = 90.45\times10^3 \angle 10.6°$ Volts per phase.

The **next step** is to calculate the sending end current. In order to do this we must first calculate the shunt current $\mathbf{I}_{shs}$ given by

$\mathbf{I}_{shs} = \mathbf{V}_s\ \mathbf{Y} = (88.9\times10^3 + j\ 16.7\times10^3)\ (j\ 0.14 \times 10^{-3}) = (-2.3 + j\ 12.4)$ A

The sending-end current $\mathbf{I}_s$ is thus given by

$\mathbf{I}_s = \mathbf{I}_{ser} + \mathbf{I}_{shs} = (393.6 - j\ 186.1) + (-2.3 + j\ 12.4) = (391.3 - j\ 173.7)$

$= 428.1 \angle -23.9°$ A.

And **finally**, the sending-end power $\mathbf{S}_s$ is given by

$S_s = V_s\ \bar{I}_s = \left(90.45\times10^3 \angle 10.6°\right)\left(428.1 \angle 23.9°\right) = 38.72\times10^6 \angle 34.5°$

$= 31.91\times10^6 + j\ 21.93\times10^6$ W and VAr per phase

or 95.73 MW and 65.8 MVAr three phase.

Note that compared with the receiving end quantities, the sending end voltage has a higher rms value (156.7 kV line-to-line voltage compared with 132 kV) and leads $\mathbf{V}_r$ by 10.6°. Also, we lose 5.73 MW and 20.8 MVAr in transmission. More surprisingly, the receiving-end current has an rms value of 440 A when the sending-end current measures only 428.1 A.

Exercise 2.3

In the above example, why is the receiving end current higher than the sending end current?

Turn to the end of the booklet for suggested answers

Exercise 2.4

Now, perform the above calculation again, this time using the short line model of Fig. 3.5. Compare your results with those obtained by using the π model and comment on the accuracy of the calculations.

Turn to the end of the booklet for suggested answers.

Summary

Just a short summary of the work we've covered in this unit:

- The electrical behaviour of power transmission lines may be modelled by relatively simple lumped parameter circuit models in which the distributed inductance, capacitance, resistance and conductance of the transmission line is represented by separate inductors, capacitors and resistors.

- The magnitude and direction of the flow of real power on a line depends on the phase angle between the sending end voltage and the receiving end voltage. Power flows from the end with the leading voltage to the end with the lagging voltage. The magnitude of power flowing down the line increases with an increasing phase difference.

- The magnitude and direction of reactive power flow on a line depends on the difference in magnitude between the sending end voltage and the receiving end voltage. Reactive power flows from the end with the higher voltage to the end with the lower voltage. The magnitude of reactive power increases with an increasing voltage difference.

- The real power losses on a line are proportional to the sum of the squares of real and reactive power flows down the line and inversely proportional to the square of the line rms voltage.