

Seminar 5: A GARCH analysis of the excess returns on the FTSE All Share Index

The objectives of this seminar are to:

- Test for ARCH effects in a series of excess returns on the FTSE All Share Index (excess market returns).
- Estimate and test a GARCH model for the excess market returns.
- Test for leverage effects and estimate asymmetric GARCH models for the excess market returns.
- Test for a time varying risk premium in the excess market returns using different versions of GARCH-in-Mean (GARCH-M) models.

The learning outcomes will be to develop your understanding of:

- Testing for ARCH effects in *Eviews*.
- Estimating GARCH models in *Eviews*.
- Misspecification testing of GARCH models and their application in *Eviews*.
- GARCH models which allow for an asymmetric response of volatility to past shocks.
- GARCH models which incorporate a time varying risk premium in the conditional mean (GARCH-M).
- Estimating and testing asymmetric GARCH and GARCH-M models in *Eviews*.

The workfile for this analysis is **ftse_all_sem5.wf1** which contains daily data on the FTSE All Share Index and the 3-month Treasury Bill rate (converted to a daily rate) for the period 1st January 2003 – 19th January 2006.

1. Background and overview

The basic GARCH model for the excess market returns can be written as follows:

$$\begin{aligned}r_{m,t} - r_f &= \mu_t + \varepsilon_t, \\ \varepsilon_t &= v_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \\ v_t &\sim NID(0,1)\end{aligned}$$

The first equation is the conditional mean equation. We will begin the analysis by assuming a constant risk premium (μ) and relax this assumption subsequently. The other three equations describe the conditional variance.

In particular, in the above specification, we've assumed that:

- i) The conditional variance follows a GARCH(1,1) model (a common assumption in empirical finance).
- ii) The conditional error distribution is Gaussian:

$$v_t \sim NID(0,1) \Rightarrow \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$$

As always, we need to test whether these assumptions relating to the statistical model hold for our data i.e., we need to carry out misspecification testing. For the GARCH model this will involve:

- Testing whether the NID assumption holds for the standardized residuals.
- Estimating and testing extensions to the basic GARCH model.

We will go through the tests on the standardized residuals below. On the second point, an important extension to the basic GARCH model is to allow for an asymmetric response of volatility to past shocks. Typically volatility may respond more to bad news (negative shocks) than good news (positive shocks) of the same magnitude. In the context of equity returns this asymmetry may be due to leverage effects: negative shocks cause the value of the firm to fall which raises the debt-equity ratio thereby increasing the risk of bankruptcy (debt-equity ratios are a key predictor of the probability of default in credit scoring models).

There are two main models which are useful for modelling asymmetric volatility: Threshold ARCH (TARCH) and Exponential GARCH (EGARCH) (see lecture 6). The asymmetry analysis begins with a general test for asymmetries in volatility called a Sign and Size Bias Test. Then, based on the evidence of this test, we will estimate and test for asymmetries in TARCH and EGARCH models.

The analysis culminates with the estimation of a time-varying risk premium model for the excess market returns. Applying CAPM to the market portfolio implies the following model for the excess market returns:

$$r_{m,t} - r_f = \lambda \sigma_t^2 + \varepsilon_t$$

where λ is the market price of risk (see Lectures 2, 6 and Cuthbertson and Nitzsche pp 137-138 and pp 659-661). The predictions of CAPM for the excess market returns are that the intercept is zero (no abnormal returns) and $\lambda > 0$ (implying a positive risk/return trade-off). In this version of the GARCH model the conditional variance enters the mean equation directly. This is known generally as a GARCH-M model. Specifically, based on the asymmetry analysis, we will estimate EGARCH-M and TARCH-M models to test CAPM.

2. Test for ARCH effects in the excess market returns

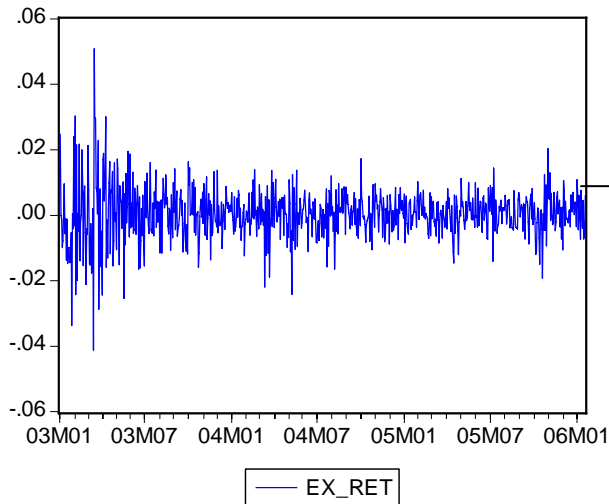
- i) Generate the excess market returns $r_{m,t} - r_f$

Click **Genr** on the workfile toolbar and enter:

```
ex_ret=dlog(ftse_all)-rf_daily
```

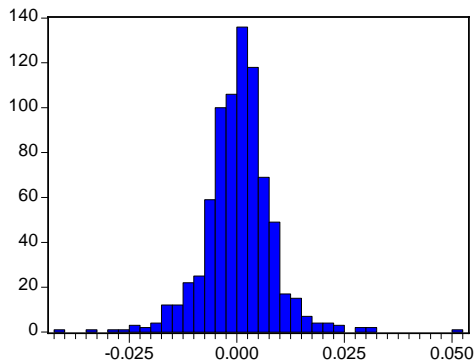
As always, reporting line graphs and summary statistics constitute an important prelude to the main analysis. Comment on the following output:

Figure 1: Line graph of ex_ret



The excess returns display periods of turbulence and tranquility. This suggests there is volatility clustering.

Table 1: Summary statistics for ex_ret



| Series: EX_RET | |
|-------------------------|----------------------|
| Sample | 1/01/2003 1/19/2006 |
| Observations | 776 |
| Mean | 0.000514 |
| Median | 0.000743 |
| Maximum | 0.050803 |
| Minimum | -0.041116 |
| Std. Dev. | 0.007912 |
| Skewness | 0.093422 |
| Kurtosis | 7.539827 |
| Jarque-Bera Probability | 667.5198 0.000000 |

The excess returns have an unconditional non-normal distribution: the distribution is leptokurtic (fat-tailed) – explain why.

Now test for ARCH effects using an ARCH-LM test:

We begin by assuming a constant risk premium and will relax this assumption later.

Alternatively, an ARMA model (identified using the ACF/PACF of ex_ret) could be used to estimate the conditional mean (see Seminar 4).

On the main toolbar click *Quick/Estimate Equation* and enter:

ex_ret c

On the Equation toolbar click *View/Residual Tests/ARCH LM Test* (Choose 5 lags)

ARCH Test:

| | | | |
|---------------|----------|---------------------|---|
| F-statistic | 49.24828 | Prob. F(5,765) | 0 |
| Obs*R-squared | 187.7416 | Prob. Chi-Square(5) | 0 |

Test Equation:
 Dependent Variable: RESID^2
 Method: Least Squares
 Date: 02/18/07 Time: 09:47
 Sample (adjusted): 1/09/2003 1/19/2006
 Included observations: 771 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|-------------|-------------|------------|-------------|--------|
| C | 2.33E-05 | 5.81E-06 | 4.013972 | 0.0001 |
| RESID^2(-1) | 0.395564 | 0.035356 | 11.18795 | 0 |
| RESID^2(-2) | 0.019361 | 0.037755 | 0.512812 | 0.6082 |
| RESID^2(-3) | 0.147726 | 0.037373 | 3.952796 | 0.0001 |
| RESID^2(-4) | -0.148994 | 0.037682 | -3.953982 | 0.0001 |
| RESID^2(-5) | 0.206817 | 0.035014 | 5.906722 | 0 |

| | | | |
|------------------|----------|-----------------------|-----------|
| R-squared | 0.243504 | Mean dependent var | 6.19E-05 |
| Adjusted R-squ: | 0.23856 | S.D. dependent var | 0.000159 |
| S.E. of regressi | 0.000139 | Akaike info criterion | -14.91586 |
| Sum squared re | 1.48E-05 | Schwarz criterion | -14.87969 |
| Log likelihood | 5756.064 | F-statistic | 49.24828 |
| Durbin-Watson : | 2.047884 | Prob(F-statistic) | 0 |

Check whether or not inferences on ARCH effects are sensitive to the chosen lag: vary the lag from one day up to 20 days (1 month).

The ARCH-LM statistic is significant at the 5% level suggesting the presence of ARCH effects. This result provides justification for the next stage in the analysis which involves estimating the conditional variance using a GARCH(1,1) model.

3. Estimate and test a GARCH(1,1) model for the conditional variance.

i) Estimate a GARCH(1,1) model

On the main toolbar click *Quick/Estimate Equation* (Method=ARCH)

In the *Mean equation* enter

ex_ret c

The default settings for the conditional variance equation are GARCH(1,1) with a conditional normal distribution (*Variance and distribution specification*). Proceed using these settings:

Dependent Variable: EX_RET
 Method: ML - ARCH (Marquardt) - Normal distribution
 Date: 02/18/07 Time: 10:12
 Sample (adjusted): 1/02/2003 1/19/2006
 Included observations: 776 after adjustments
 Convergence achieved after 13 iterations
 Variance backcast: ON
 GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|-------------|------------|-------------|--------|
| C | 0.000539 | 0.000228 | 2.361327 | 0.0182 |

Variance Equation

| | | | | |
|-------------|----------|----------|----------|--------|
| C | 7.72E-07 | 3.01E-07 | 2.565099 | 0.0103 |
| RESID(-1)^2 | 0.057111 | 0.013868 | 4.118149 | 0 |
| GARCH(-1) | 0.923931 | 0.016902 | 54.66489 | 0 |

| | | | |
|------------------|-----------|-----------------------|-----------|
| R-squared | -0.00001 | Mean dependent va | 0.000514 |
| Adjusted R-squ | -0.003896 | S.D. dependent var | 0.007912 |
| S.E. of regressi | 0.007927 | Akaike info criterion | -7.155111 |
| Sum squared re | 0.048516 | Schwarz criterion | -7.13112 |
| Log likelihood | 2780.183 | Durbin-Watson stat | 2.136473 |

Name the equation 'GARCH11' for later reference

The ARCH and GARCH coefficients (0.057 and 0.924) are statistically significant. The sum of these coefficients is 0.981 which indicates that shocks to volatility have a persistent effect on the conditional variance. These shocks will have a permanent effect if the sum of the ARCH and GARCH coefficients equals unity (\Rightarrow the conditional variance does not converge on a constant unconditional variance in the long run). In that case the model is an Integrated GARCH(1,1) – IGARCH(1,1) – see lecture 6 notes.

Test for an IGARCH model ($\alpha + \beta = 1$)

Click **View/Coefficient Tests/Wald - Coefficient Restrictions...**

$c(3)+c(4)=1$

IGARCH is rejected. Shocks to volatility are just very persistent if not permanent.

ii) Misspecification testing of the GARCH(1,1) model

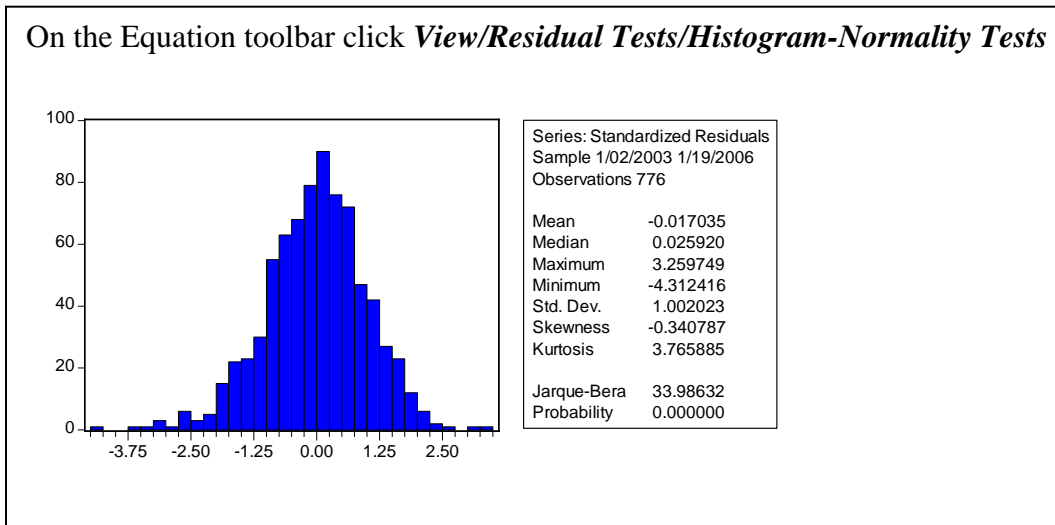
These tests are based on the (estimated) standardized residuals:

$$\hat{v}_t = \frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}$$

If the assumption $v_t \sim NID(0,1)$ is valid then:

a) The standardized residuals should be normally distributed.

On the Equation toolbar click **View/Residual Tests/Histogram-Normality Tests**



In fact, the standardized residuals in this case are non-normally distributed. However, to estimate the model using Maximum Likelihood (ML), we have assumed a conditional normal distribution:

$$v_t \sim NID(0,1) \Rightarrow \varepsilon_t | \Omega_{t-1} \sim N(0, \sigma_t^2)$$

Assuming conditional normality the likelihood function is given by:

$$\begin{aligned} \log L(\theta) &= \sum_{t=1}^T \left[\log f(v_t | \theta) - \frac{1}{2} \log(\sigma_t^2) \right] \\ &= -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log(\sigma_t^2) - \frac{1}{2} \sum_{t=1}^T \frac{\varepsilon_t^2}{\sigma_t^2} \end{aligned}$$

Choose parameter values to maximize the log-likelihood function.

If the conditional distribution is non-normal then assuming normality will still result in ML estimators which are:

- Consistent
- Asymptotically normal

However the standard errors will be inconsistent. If the conditional normality assumption is rejected then as an alternative either:

- Re-estimate the model using robust standard errors (click the options tab at the equation estimation stage and select Bollerslev-Wooldridge Heteroscedasticity Consistent Covariance Matrix). This is known as Quasi Maximum Likelihood (QML). This estimator is consistent but *not* asymptotically efficient (efficient estimator=estimator with the lowest variance).

OR

- Re-estimate the model using a different conditional error distribution (at the equation estimation stage change the ‘error distribution’ from normal to a Student’s *t* or a Generalized Error Distribution). Assuming the correct distribution has been selected, then the estimator will be consistent and asymptotically efficient (i.e., it is Exact Maximum Likelihood).

We will re-estimate the model with robust standard errors (QML). This is a ‘safe’ option: we are assured of valid (if not efficient) inferences. However, if we use Exact ML, but the alternative distributional assumptions are incorrect, then the inferences will remain invalid.

Click ***Estimate*** on the equation toolbar
Click on the options tab and check the ***Coefficient covariance*** to select a Bollerslev-Wooldridge covariance matrix

Dependent Variable: EX_RET
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 02/18/07 Time: 11:13
Sample (adjusted): 1/02/2003 1/19/2006
Included observations: 776 after adjustments
Convergence achieved after 13 iterations
Bollerslev-Wooldrige robust standard errors & covariance
Variance backcast: ON
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|-------------|------------|-------------|--------|
| C | 0.000539 | 0.000216 | 2.489807 | 0.0128 |

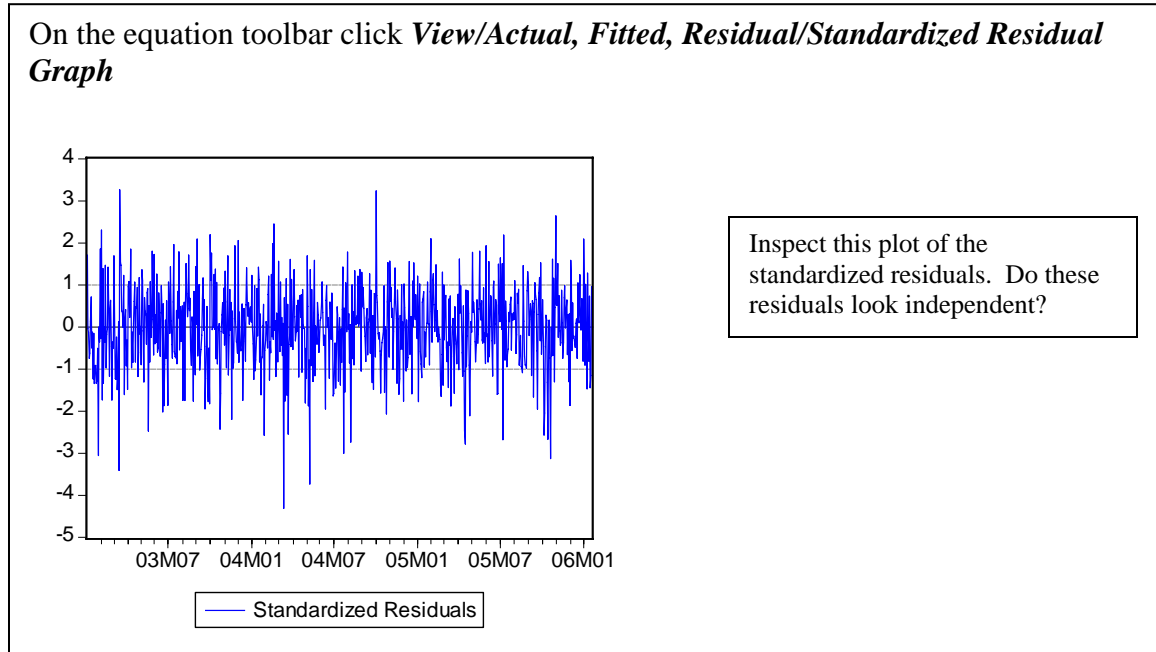
| Variance Equation | | | | |
|-------------------|----------|----------|----------|--------|
| C | 7.72E-07 | 3.16E-07 | 2.44352 | 0.0145 |
| RESID(-1)^2 | 0.057111 | 0.020328 | 2.809516 | 0.005 |
| GARCH(-1) | 0.923931 | 0.022426 | 41.1987 | 0 |

| | | | |
|------------------|-----------|-----------------------|-----------|
| R-squared | -0.00001 | Mean dependent va | 0.000514 |
| Adjusted R-squ | -0.003896 | S.D. dependent var | 0.007912 |
| S.E. of regressi | 0.007927 | Akaike info criterion | -7.155111 |
| Sum squared re | 0.048516 | Schwarz criterion | -7.13112 |
| Log likelihood | 2780.183 | Durbin-Watson stat | 2.136473 |

Now that we have re-specified the model to take account of conditional-normality, compare inferences between the original GARCH model and the ‘robust’ model. The standard errors and *p*-values are different. This could make a crucial difference to the inferences (although it doesn’t in this case).

There are other misspecification tests that need to be carried out. A second implication of the assumption $v_t \sim NID(0,1)$ is that:

b) The standardized residuals should be independent.



To help you reach a firmer conclusion you can conduct more formal tests of independence....

On the equation toolbar click **View/Residual Tests/Correlogram Q Statistics**

| Autocorrel | Partial Correlation | AC | PAC | Q-Stat | Prob | |
|------------|---------------------|----|--------|--------|--------|-------|
| . | . | 1 | -0.056 | -0.056 | 2.4825 | 0.115 |
| . | . | 2 | 0.058 | 0.055 | 5.1367 | 0.077 |
| . | . | 3 | -0.016 | -0.01 | 5.3357 | 0.149 |
| . | . | 4 | 0.022 | 0.018 | 5.7153 | 0.221 |
| . | . | 5 | -0.012 | -0.009 | 5.8326 | 0.323 |
| . | . | 6 | -0.005 | -0.008 | 5.8518 | 0.44 |
| . | . | 7 | -0.039 | -0.039 | 7.0722 | 0.421 |
| . | . | 8 | 0.021 | 0.017 | 7.4213 | 0.492 |
| . | . | 9 | -0.031 | -0.024 | 8.1568 | 0.518 |
| . | . | 10 | -0.019 | -0.025 | 8.4538 | 0.585 |

This is the ACF/PACF for the standardized residuals. In this case, the Q tests suggest these residuals are linearly independent (be sure you understand why).

and...

On the equation toolbar click **View/Residual Tests/Correlogram Squared Residuals**

| Autocorrel | Partial Correlation | AC | PAC | Q-Stat | Prob | |
|------------|---------------------|----|--------|--------|--------|-------|
| . . | . . | 1 | 0.024 | 0.024 | 0.4393 | 0.507 |
| . . | . . | 2 | 0.004 | 0.003 | 0.4501 | 0.798 |
| . . | . . | 3 | 0.003 | 0.003 | 0.4562 | 0.928 |
| . . | . . | 4 | -0.024 | -0.024 | 0.9092 | 0.923 |
| . * | . * | 5 | 0.068 | 0.069 | 4.4892 | 0.481 |
| . . | . . | 6 | 0.011 | 0.008 | 4.5798 | 0.599 |
| . . | . . | 7 | 0.057 | 0.057 | 7.1358 | 0.415 |
| . . | . . | 8 | 0.032 | 0.028 | 7.9457 | 0.439 |
| . . | . . | 9 | -0.024 | -0.022 | 8.3945 | 0.495 |
| . . | . . | 10 | -0.023 | -0.027 | 8.8138 | 0.55 |

This is the ACF/PACF for the standardized residuals squared. Here the Q test suggests the standardized residuals are also non-linearly independent.

These correlograms and Q statistics support the hypothesis that the standardized residuals are independent. These tests therefore suggest that the GARCH(1,1) model (with robust Bollerslev-Wooldridge standard errors) is well specified: so far so good.

However, the GARCH model assumes a symmetric response of volatility to past shocks. In the context of stock market returns, there is the possibility of leverage effects. In that case a negative shock (reducing the stock market value of the business) will raise the debt-equity ratio so that the firm will appear more risky to investors. As a consequence negative shocks will raise volatility more than a positive shock of equal magnitude. This leads to the possibility of asymmetries in volatility...

4. Testing for asymmetries in volatility (see Brooks Chp 8.16)

To test whether (past) positive and negative shocks have a different impact on volatility a Sign Bias (SB) Test can be carried out:

Step 1: Obtain the residuals from the (symmetric) GARCH model.

Step 2: Test for sign bias in the following regression of the squared residuals:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 S_{t-1}^- + u_t,$$

$$S_{t-1}^- = 1 \text{ if } \hat{\varepsilon}_{t-1} < 0 \text{ and } S_{t-1}^- = 0 \text{ otherwise}$$

The test for sign bias involves a t-test on the coefficient α_1 : if positive and negative shocks have different impacts on volatility then α_1 will be statistically significant.

A more general test involves testing whether volatility depends on both the sign and size of past shocks. This is a Sign and Size Bias Test. This test is based on the following regression:

$$\hat{\epsilon}_t^2 = \alpha_0 + \alpha_1 S_{t-1}^- + \alpha_2 S_{t-1}^- \hat{\epsilon}_{t-1} + \alpha_3 S_{t-1}^+ \hat{\epsilon}_{t-1} + u_t,$$

$$S_{t-1}^+ = 1 - S_{t-1}^-$$

The null hypothesis of no sign and size bias corresponds to: $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = 0$. This can be tested with a Lagrange Multiplier (LM) test. Under the null hypothesis:

$$TR^2 \overset{a}{\sim} \chi^2(3).$$

We'll carry out the Sign and Size Bias test. Firstly obtain the residuals from the GARCH model:

On the equation toolbar click **Procs/Make Residual Series**. Name the residuals 'resid_garch'

Then create the dummy variable for negative shocks:

On the workfile toolbar click **Genr** and enter:
s=resid_garch<0

Now run the sign and size bias test regression:

On the main toolbar click **Quick/Estimate Equation** (Method=Least Squares):

resid_garch^2 c s(-1) s(-1)*resid_garch(-1) (1-s(-1))*resid_garch(-1)

Dependent Variable: RESID_GARCH^2
Method: Least Squares
Date: 02/18/07 Time: 15:19
Sample (adjusted): 1/03/2003 1/19/2006
Included observations: 775 after adjustments

| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
|---------------------------|-------------|-----------------------|-------------|--------|
| C | 1.50E-05 | 1.06E-05 | 1.410174 | 0.1589 |
| S(-1) | -2.18E-06 | 1.55E-05 | -0.140744 | 0.8881 |
| S(-1)*RESID_GARCH(-1) | -0.008954 | 0.001395 | -6.419375 | 0 |
| (1-S(-1))*RESID_GARCH(-1) | 0.008134 | 0.001373 | 5.925781 | 0 |
| R-squared | 0.09038 | Mean dependent va | 6.18E-05 | |
| Adjusted R-squared | 0.086841 | S.D. dependent var | 0.000159 | |
| S.E. of regression | 0.000152 | Akaike info criterion | -14.74159 | |
| Sum squared resid | 1.78E-05 | Schwarz criterion | -14.71757 | |
| Log likelihood | 5716.365 | F-statistic | 25.53555 | |
| Durbin-Watson stat | 1.720522 | Prob(F-statistic) | 0 | |

Name this equation sb test

Now calculate the LM test and p -value.

In the command window type in:

```
scalar lm=sb_test.@regobs*sb_test.@r2
scalar lm_pval=1-@cchisq(lm,3)
```

Double click on the scalar objects and look at the bottom of the *Eviews* window to see their values.

The p -value for the test indicates that the null of no sign and size bias is strongly rejected. We will therefore estimate the TARCh and EGARCH models which allow for an asymmetric response of volatility to past shocks. Firstly estimate the TARCh model

Open the equation 'GARCH11'. Click on the *Estimate* button; change the *Threshold order* to 1 and click OK

Dependent Variable: EX_RET
 Method: ML - ARCH (Marquardt) - Normal distribution
 Date: 02/18/07 Time: 15:41
 Sample (adjusted): 1/02/2003 1/19/2006
 Included observations: 776 after adjustments
 Convergence achieved after 16 iterations
 Bollerslev-Wooldrige robust standard errors & covariance
 Variance backcast: ON
 GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0)
 + C(5)*GARCH(-1)

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|-------------|------------|-------------|--------|
| C | 0.000329 | 0.000214 | 1.537198 | 0.1242 |

Variance Equation

| | Coefficient | Std. Error | z-Statistic | Prob. |
|-------------------------------------|-----------------|-----------------|-----------------|---------------|
| C | 9.69E-07 | 3.49E-07 | 2.77939 | 0.0054 |
| RESID(-1)^2 | -0.005338 | 0.019971 | -0.267307 | 0.7892 |
| RESID(-1)^2*(RESID(-1)<0) | 0.116697 | 0.037636 | 3.100672 | 0.0019 |
| GARCH(-1) | 0.923517 | 0.019471 | 47.43031 | 0 |

| | | | |
|--------------------|-----------|--------------------------|------------------|
| R-squared | -0.000545 | Mean dependent var | 0.000514 |
| Adjusted R-squared | -0.005736 | S.D. dependent var | 0.007912 |
| S.E. of regression | 0.007935 | Akaike info criterion | -7.175475 |
| Sum squared resid | 0.048542 | Schwarz criterion | -7.145487 |
| Log likelihood | 2789.084 | Durbin-Watson stat | 2.13533 |

This is a TARCh(1,1) model. The coefficient on the asymmetry term is positive and statistically significant. This supports the presence of leverage effects.

Freeze this view of the equation and name the resulting table object: TARCh11

Now estimate the EGARCH model

Open GARCH11 and click the *Estimate* button. Change the *Model* to EGARCH and click OK

Dependent Variable: EX_RET
 Method: ML - ARCH (Marquardt) - Normal distribution
 Date: 02/18/07 Time: 15:51
 Sample (adjusted): 1/02/2003 1/19/2006
 Included observations: 776 after adjustments
 Convergence achieved after 12 iterations
 Bollerslev-Wooldrige robust standard errors & covariance
 Variance backcast: ON
 LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
 C(4)*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|-------------|------------|-------------|--------|
| C | 0.000378 | 0.000212 | 1.777906 | 0.0754 |

Variance Equation

| | | | | |
|-------------|------------------|-----------------|------------------|---------------|
| C(2) | -0.120229 | 0.044075 | -2.727828 | 0.0064 |
| C(3) | 0.04865 | 0.022572 | 2.155312 | 0.0311 |
| C(4) | -0.082004 | 0.021597 | -3.796969 | 0.0001 |
| C(5) | 0.992026 | 0.003482 | 284.8753 | 0 |

| | | | |
|--------------|-----------|--------------------------|------------------|
| R-squared | -0.000297 | Mean dependent var | 0.000514 |
| Adjusted R | -0.005487 | S.D. dependent var | 0.007912 |
| S.E. of reg | 0.007934 | Akaike info criterion | -7.17731 |
| Sum squar | 0.04853 | Schwarz criterion | -7.147322 |
| Log likelihc | 2789.796 | Durbin-Watson stat | 2.135859 |

The coefficient on the asymmetry term, C(4), is negative and statistically significant. Again this supports the presence of leverage effects.

Name this equation EGARCH11

Which asymmetric model is preferred: TARARCH or EGARCH? The Schwarz criterion is *marginally* smaller for the EGARCH model (which suggests EGARCH is the better model) - but there's so little to choose between the two models that we will use both in the final piece of analysis.

5. GARCH-in-Mean (GARCH-M)

Applying CAPM to the market portfolio implies the following model for the excess market returns:

$$r_{m,t} - r_f = \lambda \sigma_t^2 + \varepsilon_t$$

where λ is the market price of risk (see Lectures 2, 6 and Cuthbertson and Nitzsche pp 137-138 and pp 659-661). The conditional variance enters the mean equation directly implying a time-varying risk premium. This is known generally as a GARCH-M model. Specifically we will estimate EGARCH-M and TARARCH-M models building on the analysis of asymmetric volatility in section 3.

Begin by estimating an EGARCH-M model:

Open the equation EGARCH11 and click on the *Estimate* button. Click on *ARCH-M* and select *Variance*. Then click OK:

Dependent Variable: EX_RET
 Method: ML - ARCH (Marquardt) - Normal distribution
 Date: 02/18/07 Time: 16:08
 Sample (adjusted): 1/02/2003 1/19/2006
 Included observations: 776 after adjustments
 Convergence achieved after 16 iterations
 Bollerslev-Wooldrige robust standard errors & covariance
 Variance backcast: ON

$$\text{LOG}(\text{GARCH}) = \text{C}(3) + \text{C}(4) \cdot \text{ABS}(\text{RESID}(-1) / \sqrt{\text{GARCH}(-1)}) + \text{C}(5) \cdot \text{RESID}(-1) / \sqrt{\text{GARCH}(-1)} + \text{C}(6) \cdot \text{LOG}(\text{GARCH}(-1))$$

| | Coefficient | Std. Error | z-Statistic | Prob. |
|--------------|----------------|-----------------|-----------------|---------------|
| GARCH | 13.0795 | 8.155953 | 1.603676 | 0.1088 |
| C | -0.00012 | 0.000377 | -0.316964 | 0.7513 |

The EGARCH-M term is statistically insignificant suggesting there is no risk/return trade-off ⇒ CAPM is rejected.

Variance Equation

| | | | | |
|------|-----------|----------|-----------|--------|
| C(3) | -0.197456 | 0.071347 | -2.76754 | 0.0056 |
| C(4) | 0.048463 | 0.022392 | 2.164259 | 0.0304 |
| C(5) | -0.084591 | 0.020434 | -4.139628 | 0 |
| C(6) | 0.984433 | 0.006543 | 150.4486 | 0 |

| | | | |
|--------------|-----------|--------------------------|------------------|
| R-squared | 0.001353 | Mean dependent var | 0.000514 |
| Adjusted R | -0.005132 | S.D. dependent var | 0.007912 |
| S.E. of reg | 0.007932 | Akaike info criterion | -7.178554 |
| Sum squar | 0.04845 | Schwarz criterion | -7.142568 |
| Log likelihc | 2791.279 | F-statistic | 0.208671 |
| Durbin-Wa | 2.111499 | Prob(F-statistic) | 0.958882 |

Finish the analysis by estimating a TAR-ARCH-M model.

Click the *Estimate* button on the equation toolbar. Change the *Model* to TAR-ARCH and click OK

Dependent Variable: EX_RET
 Method: ML - ARCH (Marquardt) - Normal distribution
 Date: 02/18/07 Time: 16:20
 Sample (adjusted): 1/02/2003 1/19/2006
 Included observations: 776 after adjustments
 Convergence achieved after 21 iterations
 Bollerslev-Wooldrige robust standard errors & covariance
 Variance backcast: ON
 GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0)
 + C(6)*GARCH(-1)

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---------------------------|-----------------|--------------------------|------------------|--------------|
| GARCH | 17.70176 | 6.307223 | 2.806586 | 0.005 |
| C | -0.000327 | 0.000303 | -1.079676 | 0.2803 |
| Variance Equation | | | | |
| C | 1.49E-06 | 8.66E-07 | 1.719052 | 0.0856 |
| RESID(-1)^2 | -0.023541 | 0.019372 | -1.215222 | 0.2243 |
| RESID(-1)^2*(RESID(-1)<0) | 0.129743 | 0.033818 | 3.836509 | 0.0001 |
| GARCH(-1) | 0.918231 | 0.017719 | 51.82098 | 0 |
| R-squared | 0.009728 | Mean dependent var | 0.000514 | |
| Adjusted R-squared | 0.003298 | S.D. dependent var | 0.007912 | |
| S.E. of regression | 0.007899 | Akaike info criterion | -7.179563 | |
| Sum squared resid | 0.048044 | Schwarz criterion | -7.143577 | |
| Log likelihood | 2791.67 | F-statistic | 1.512863 | |
| Durbin-Watson stat | 2.104188 | Prob(F-statistic) | 0.183346 | |

The intercept is statistically insignificant and the TAR-ARCH-M coefficient is positive and statistically significant. This supports the CAPM model ⇒ no abnormal returns and a positive risk/return trade-off.

Compare the Schwarz criteria for the EGARCH-M and TAR-ARCH-M models – this one is smaller. This supports the use of the TAR-ARCH-M specification to test CAPM.

If you were doing a similar analysis for a dissertation then, before reporting the final results from the TAR-ARCH-M model, it would be wise to repeat the misspecification tests (as carried out in section 3). The first (and only) ‘law’ of econometrics is: TEST, TEST, TEST. Carry out these tests in your own time to develop your understanding of misspecification testing in the context of GARCH models.

Conclusion

GARCH models provide a rich approach to modelling time varying volatility. There are numerous variants on the ‘plain vanilla’ GARCH model; those presented in this seminar handout are examples of the most popular models. Building on a simple GARCH model, we were able to incorporate leverage effects through the use of asymmetric GARCH models (TAR-ARCH/EGARCH). Extending this further to a TAR-ARCH-M/EGARCH-M framework allowed us to test the predictions of CAPM for the market returns. These predictions (no abnormal returns and a positive risk/return trade-off) were not rejected in the TAR-ARCH-M model.