WARWICK BUSINESS SCHOOL

Empirical Finance
Lecture 2: Tests of the EMH

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Last time…

- Price/return series (e.g., FTSE 250,) are not well modelled with the RWM:
  - Prices are non-stationary but…
  - Returns are *not* independent:
    - Volatility clustering ⇒ non-linear dependence.
    - Are they linearly independent? (today's topic).
  - Returns are (very) non-Gaussian.

Today…

- A less restrictive version of the RWM.
  - The martingale model (MM).
- Tests of return predictability.
  - ACFs
  - Regression based tests
  - Variance ratio tests
- The interpretation of evidence for return predictability.
  - Critical appraisal of the relationship between the RWM/MM and EMH.

Martingale Model

- There are 2 main objections to the RWM as a DGP for financial data:
  - Assumption of independence of returns.
  - Assumption of normality of returns.
- The MM is similar to the RWM but assumes only that returns are linearly independent.
- The MM therefore provides a better description of price movements under the EMH.

Martingale model (Cuthbertson and Nitzsche 3.3)

$\{x_t\}$ is a martingale process if:

i) $E(|x_t|) < \infty$ (the mean is bounded)

ii) $E(x_{t+h} | \Omega_t) = x_t, \quad h > 0$

Martingale property

A martingale is a model of a fair game

$$E(x_{t+h} - x_t | \Omega_t) = 0$$

The expected h-period return is zero.

Example of a fair game: A game of tossing an unbiased coin:

win £1 for a head; lose £1 for a tail

The expected return is £0 per play:

E(r)=1×0.5 –1 ×0.5=0

The process is a sub-martingale

if $E(x_{t+h} - x_t | \Omega_t) > 0$

It is a super-martingale if

$E(x_{t+h} - x_t | \Omega_t) < 0$

Martingale model

We can write an equation for x which looks rather like the RWM:

$$x_t = x_{t-1} + \varepsilon_t$$

If the process is a sub- or super- martingale then Include a drift term in the equation

$$x_t = \mu + x_{t-1} + \varepsilon_t$$

where ε is a martingale difference (or increment).

The fair game property means that the best guess of a future increment is that it equals zero.

$$E\left(\varepsilon_{t+h} \middle| \Omega_t\right) = 0, \quad h > 0$$

However there is no assumption that the increments are

1. Independent (fair game property ⇒linear independence only).
2. Normally distributed.

Indeed neither independence nor a Gaussian distribution is required for returns under the EMH (only linear independence).

MM and the EMH (Cuthbertson & Nitzsche 3.1-3.4)

- EMH states that asset prices fully reflect all available relevant information:
- The only systematic/predictable gain (change in price) is the required rate of return on the asset. Other gains/losses are attributable to unpredictable events: news
  - $\Rightarrow$Investors cannot make abnormal profits systematically from buying and selling assets: risk adjusted returns are a fair game

Investors form rational expectations:
i) They know Ω_t and the true model for returns; and
ii) They use this information to predict future returns

$$E\left(r_{t+1} \middle| \Omega_t\right) - \mu = E\left(\varepsilon_{t+1} \middle| \Omega_t\right) = 0$$

RE implies forecast errors are unpredictable given Ω_t

- The key empirical prediction of the MM/EMH is that future returns are linearly independent from information available in the current or previous periods.

Empirical testing of EMH

EMH comes in 3 flavours depending on what constitutes the relevant information set:

1. Weak form efficiency – current price incorporates all the information on past prices.
2. Semi strong efficiency – current price incorporates *all* publicly available information.
3. Strong form efficiency– current price incorporates *all* information including insider information.

Tests of EMH are usually of the weak or semi-strong form variety.

Tests of weak/semi-strong form EMH (Brooks 5.2)

- Analysis of the ACF of returns

Implied by linear independence

$$\operatorname{cov}(r_t, r_{t-s}) = 0, \ s > 0$$
$$\Rightarrow \rho_s = \frac{\operatorname{cov}(r_t, r_{t-s})}{\operatorname{var}(r_t)} = 1, \quad s = 0$$
$$= 0, \ s > 0$$

The population ACF of a linearly independent process is flat at zero for all positive lags.

Risk adjusted returns have exactly the same ACF as here (assuming constant equilibrium returns).

A mean zero, linearly independent process (such as risk adjusted returns) is a WHITE NOISE process.

White noise has an ACF with the shape given here.

- In any given sample $\hat{\rho}_s \neq 0$ even if $\rho_s = 0$
- The null hypothesis...

$$H_0 : \rho_1 = \rho_2 = \ldots = \rho_k = 0$$

- ...can be tested with the Ljung-Box Q statistic ('portmanteau' statistic):

$$Q(k) = T(T+2)\sum_{i=1}^{k}(T-i)^{-1}\hat{\rho}_i^2 \overset{a}{\sim} \chi_k^2$$

Large sample/ asymptotic distribution

ACFs of return series

FTSE 250 returns

<table>
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<tr><td>Autocorrel</td><td>Partial Cor</td><td>Lag</td><td>AC</td><td>PAC</td><td>Q-Stat</td><td>Prob</td></tr>
<tr><td>|**</td><td>|**</td><td>1</td><td>0.216</td><td>0.216</td><td>236.19</td><td>0</td></tr>
<tr><td>|*</td><td>|*</td><td>2</td><td>0.11</td><td>0.067</td><td>297.75</td><td>0</td></tr>
<tr><td>|*</td><td>|</td><td>3</td><td>0.078</td><td>0.044</td><td>328.66</td><td>0</td></tr>
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<tr><td>|</td><td>|</td><td>6</td><td>0.016</td><td>-0.013</td><td>421.36</td><td>0</td></tr>
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<tr><td>|*</td><td>|</td><td>8</td><td>0.076</td><td>0.045</td><td>468.85</td><td>0</td></tr>
<tr><td>|*</td><td>|</td><td>9</td><td>0.077</td><td>0.045</td><td>498.53</td><td>0</td></tr>
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<tr><td>|</td><td>|</td><td>15</td><td>0.029</td><td>0.011</td><td>542.22</td><td>0</td></tr>
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Linear independence/EMH rejected

UK-US exchange rate returns

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<tr><td>.|.</td><td>.|.</td><td>1</td><td>0.062</td><td>0.062</td><td>1.4991</td><td>0.221</td></tr>
<tr><td>.|.</td><td>.|.</td><td>2</td><td>0.008</td><td>0.005</td><td>1.527</td><td>0.466</td></tr>
<tr><td>.|.</td><td>.|.</td><td>3</td><td>-0.002</td><td>-0.003</td><td>1.5284</td><td>0.676</td></tr>
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<tr><td>*|.</td><td>.|.</td><td>15</td><td>-0.075</td><td>-0.047</td><td>21.908</td><td>0.11</td></tr>
</table>

Linear independence/EMH cannot be rejected

Tests of semi-strong EMH (Cuthbertson & Nitzsche 4.2-4.3)

Regression tests of return predictability:

$$r_{t+1} = \mu + \beta'\Omega_t + u_{t+1}$$

If excess returns are a fair game, $E_t(r_{t+1}) - \mu = 0$, then the elements of the k×1 vector of coefficients β are jointly zero.

A test of EMH is therefore:

$$H_0 : \beta_1 = \beta_2 = ... = \beta_k = 0 \text{ versus}$$
$$H_1 : \text{At least one of the } \beta' s \neq 0.$$

This can be tested using an *F*-test (see Gujarati Chp 8/ Brooks Chp 3):

$$F = \frac{ESS / k}{RSS / [T - (k+1)]} \sim F_{k, T-(k+1)} \text{ under } H_0.$$

Regression tests of return predictability

What variables are included in Ω ?

1. Data on past returns (another test of 'weak form' efficiency).
2. Data on past news $\varepsilon_{t-j},\ j \geq 0$: Autoregressive moving average models.
3. Data on other financial variables such as dividend yields, E/P and interest rates (see Fama, 1991, for a discussion).
4. Data related to various 'anomalies' (e.g., small firm and calendar effects: see Cuthbertson & Nitzsche 18.4).

A fifth order autoregression with calendar effects for FTSE250 returns

$$\hat{r}_t = \hat{\mu} + \hat{\beta}_1 MONDAY + \hat{\beta}_2 JANUARY + \hat{\beta}_3 r_{t-1} + \hat{\beta}_4 r_{t-2} + \hat{\beta}_5 r_{t-3} + \hat{\beta}_6 r_{t-4} + \hat{\beta}_7 r_{t-5}$$

Dependent Variable: FTSE_250_RETURNS				
Method: Least Squares				
Sample (adjusted): 1/09/1986 1/05/2006				
Included observations: 5046 after adjustments				
Convergence achieved after 5 iterations				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000541	0.00018	3.001562	0.0027
MONDAY	-0.00051	0.000249	-2.04459	0.0409
JANUARY	0.000688	0.000561	1.225583	0.2204
AR(1)	0.1948	0.014096	13.81955	0
AR(2)	0.052071	0.014286	3.644843	0.0003
AR(3)	0.023881	0.014304	1.669536	0.0951
AR(4)	0.098634	0.014294	6.900143	0
AR(5)	-0.00293	0.014096	-0.20787	0.8353
R-squared	0.063025	Mean dependent va		0.000503
Adjusted R	0.061723	S.D. dependent var		0.007797
S.E. of reg	0.007552	Akaike info criterion		-6.9324
Sum squar	0.287338	Schwarz criterion		-6.92205
Log likelih	17498.45	F-statistic		48.41123
Durbin-Wa	1.99968	Prob(F-statistic)		0

MONDAY=1 if DoW is Monday
=0 otherwise
JANUARY=1 if Month is January
=0 otherwise

Coefficients on $r_{t-1}, \ldots, r_{t-5}$

Semi-strong EMH rejected for FTSE250 returns

Interpreting these results

- These results indicate a violation of informational efficiency for the FTSE250.
  - For example, a trading strategy of short-selling the index on Friday and repurchasing on Monday yields a predictable return of 0.05%
- However this need not imply that investors can derive persistent abnormal profits in the ‘real world’:
- 1. Buying/selling shares incurs transactions costs.
  - Transactions costs of 5p per £100 traded would wipe out profits from the above trading strategy.
  - A flat rate fee of £10 per trade (competitive against e.g., TD Waterhouse) would require a transaction larger than £20K to start making a profit. This is risky! (see below)
- 2. Low R^2 implies potential arbitrage opportunities are very risky:
  - Only 6% of the total variation in returns is explained by the model.
  - Actual returns could be much higher or lower than expected.
- 3. The relationship may be unstable over time
  - Test the model for parameter stability before betting your house/career on your trading strategy.

Short vs long horizon returns

Positive autocorrelations in the short run may be due to noise traders

These traders (irrationally) interpret random price changes as containing information which can be used to forecast future changes .

A positive feedback trader is a noise trader who believes that a random price rise today is indicative of a price rise tomorrow.

The smart money, on the other hand, knows that price movements are random and are useless for forecasting.

Following a price rise positive feedback traders bid up the price of the stock yet further driving the price further away from fundamentals (equilibrium level)

$\Rightarrow$ positive autocorrelation over short horizons (e.g., <1 year).

Eventually though the bubble bursts:

Prices/returns go back to their equilibrium level:

mean reversion.

$\Rightarrow$negative autocorrelation over longer horizons (e.g., 1 year or more).

Variance ratio tests

These tests are widely used to test for mean reversion.

They involve comparisons of return variances for different holding periods.

k-period holding returns are given by:

$$r_{t,t+k} = \ln P_{t+k} - \ln P_t = k\mu + \varepsilon_{t+1} + \varepsilon_{t+2} + \ldots + \varepsilon_{t+k}$$
$$E\left(r_{t,t+k} \middle| \Omega_t\right) = k\mu,$$
$$\mathrm{var}\left(r_{t,t+k}\right) = k\sigma^2$$

Assuming returns are linearly independent with constant variance then risk and return Increase with the holding period.

The variance ratio statistic is defined:

$$VR_k = \frac{1}{k}\frac{\mathrm{var}\left(r_{t,t+k}\right)}{\mathrm{var}\left(r_{t+1}\right)} = 1 + \frac{2}{k}\sum_{j=1}^{k-1}(k-j)\rho_j$$

If returns are linearly independent then $VR_k = 1$ at all horizons.

Mean aversion ⇒positive autocorrelation in returns ⇒ $VR_k > 1$

Mean reversion ⇒negative autocorrelation in returns ⇒ $VR_k < 1$

Poterba and Summers (1988) found US stock returns displayed mean aversion for k<1year and mean reversion for k>1 year

Variance ratio tests

- A test for $H_0 : VR_k = 1$ is given by (see Cuthbertson & Nitzsche 4.3):

$$Z_k = \frac{VR_k - 1}{\sqrt{\frac{2(2k-1)(k-1)}{3kT}}} \overset{a}{\sim} N(0,1)$$

This test assumes returns are independent Gaussian with constant variance under the null.

Adjusted versions of the test allow for non-normality and heteroscedasticity

- VR test results for FTSE250 returns:

k	50 days	100 days	200 days	500 days
VR_k	2.590	2.439	2.096	1.539
$Z_k\,(p-value)$	0.000	0.000	0.000	0.138

There is substantial mean aversion for horizons of under a year.

There is no evidence for mean reversion at any horizon: returns move away from equilibrium and never return.

Other interpretations of the results (when is a correlation not evidence against the EMH?)

It's possible that return predictability is not a symptom of market inefficiency at all!

Instead, it may be a symptom of:

1. A poor model of equilibrium returns

Or a market microstructure issue e.g.,

2. Bid/ask bounce
3. Infrequent or non-synchronous trading

1. A poor model of equilibrium returns

Only able to test EMH conditional on a specific model of equilibrium returns (so far we have simply assumed μ is a constant – no structure)

There is a 'joint hypothesis problem'

'As a result, when we find anomalous evidence on the behavior of returns, the way it should be split between market inefficiency or a bad model of market equilibrium is ambiguous.' Fama (1991) p 1576

Even possible for returns to be predictable even when markets are informationally efficient! (Leroy, 1973)

(MM$\Leftrightarrow$EMH only under risk neutrality)

Illustration of joint hypothesis problem

Suppose the true model of returns is

$$r_{t+1} = r_f + \lambda\sigma_{t+1}^2 + \varepsilon_{t+1}$$

This equation follows by applying CAPM to the market portfolio

$$\begin{aligned} E(r) - r_f &= \beta\left[E(r_m) - r_f\right] \\ &= \frac{\text{cov}(r, r_m)}{\sigma_m^2}\left[E(r_m) - r_f\right] \\ &= \lambda\,\text{cov}(r, r_m) \\ &= \lambda\sigma_m^2, \quad r \equiv r_m \end{aligned}$$

λ Is the MARKET PRICE OF RISK :

$$\left(E(r_m) - r_f\right)/\sigma_m^2$$

where ε is a martingale difference.

Volatility clustering (see lecture 1) implies risk is time-varying and predictable.

The GARCH(1,1) model is commonly used in finance to model volatility clustering

Conditional variance

$$\sigma_{t+1}^2 = \delta + \alpha\sigma_t^2 + \beta\varepsilon_t^2$$

Illustration of joint hypothesis problem

The CAPM model plus GARCH means r_{t+1} is predictable because:

i) investors are risk averse ($\lambda>0$).

ii) risk today is a predictor of risk tomorrow (volatility clustering).

But, *conditional* on investors' preferences, price movements are unpredictable (i.e., the ε satisfy the EMH).

Illustration of joint hypothesis problem

If we mistakenly assumed (log) prices followed a (sub) martingale model (MM)

$$r_{t+1} = \mu + \varepsilon_{t+1}$$

then it would appear that EMH is violated because we have not taken into account:

i) risk aversion

ii) volatility clustering.

As a consequence, the time varying risk premium is subsumed in ε so that returns are predictable.

But the problem is a poor model of equilibrium returns rather than violation of EMH

Note that if investors are risk neutral (λ=0) then MM$\Leftrightarrow$EMH.

2. Bid-ask bounce

Market makers:

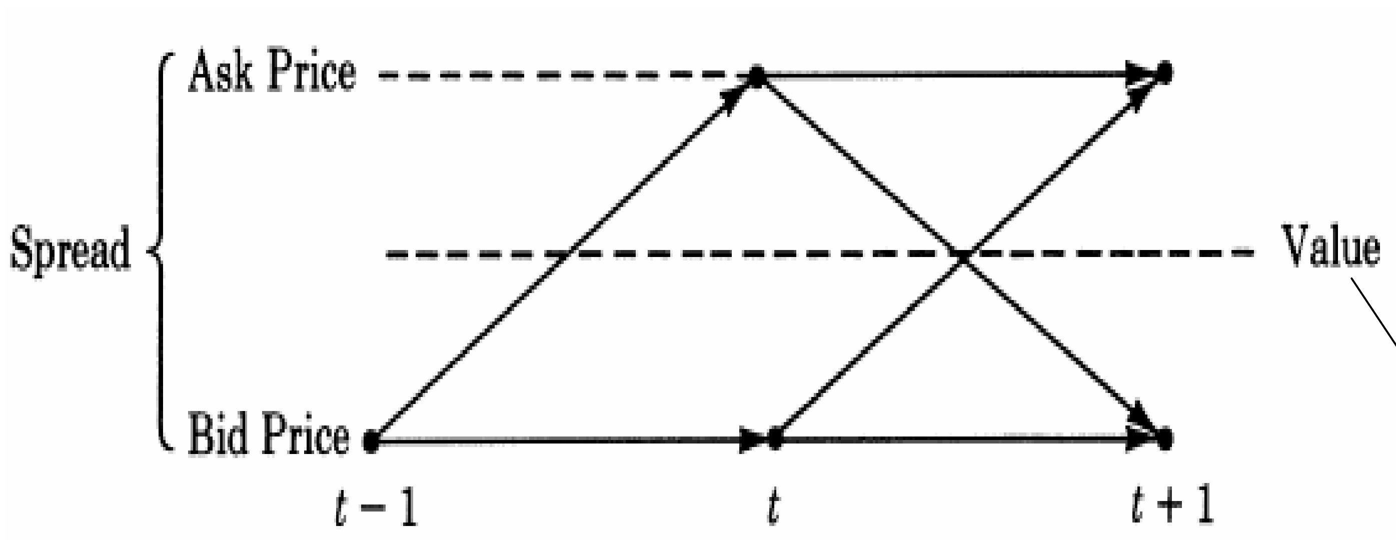
- Buy stocks from the public at a *bid* price P_b
- Sell stocks to the public at an *ask* price P_a

The bid-ask spread $S \equiv P_a - P_b$ reflects order processing costs, inventory costs and adverse selection costs

Recorded transactions could be at either the bid or ask price.

This gives rise to spurious negative autocorrelation in returns as a result of the recorded transaction 'bouncing' between the bid and ask price (see next slide).

2. Bid-ask bounce



P^*

Transaction price is given by:

$$P_t = P_t^* + I_t \frac{S}{2}$$

Indicator function

$I_t = 1$ with prob $= 0.5$ (transaction at the ask price)

$I_t = -1$ with prob $= 0.5$ (transaction at the bid price)

2. Bid-ask bounce causes spurious negative autocorrelation in price movements

- To show this need to derive the mean, variance and first order covariance of ΔI_t
- Firstly the probability distribution of ΔI_t is:

$$\text{Ask to bid}: \Delta I_t = 1 - -1 = 2 \text{ with prob} = 0.25$$
$$\text{Ask to ask}: \Delta I_t = 1 - 1 = 0 \text{ with prob} = 0.25$$
$$\text{Bid to ask}: \Delta I_t = -1 - 1 = -2 \text{ with prob} = 0.25$$
$$\text{Bid to bid}: \Delta I_t = -1 - -1 = 0 \text{ with prob} = 0.25$$

$$E(\Delta I_t) = 2\times 0.25 - 2\times 0.25 = 0$$
$$\text{var}(\Delta I_t) = E(\Delta I_t^2) = 4\times 0.25 + 4\times 0.25 = 2$$
$$\text{cov}(\Delta I_t, \Delta I_{t-1}) = E(\Delta I_t \Delta I_{t-1}) = -E(I_{t-1}^2) = -1$$

$$E[(I_t - I_{t-1})(I_{t-1} - I_{t-2})]$$
$= -E(I_{t-1}^2)$ assuming I is linearly independent.

Also, from the previous slide $E(I_{t-1}^2) = 1\times 0.5 + 1\times 0.5 = 1$

2. Bid-ask bounce causes spurious negative autocorrelation in price movements

Assume the underlying share value P_t^* follows a martingale

$$\Delta P_t^* = \varepsilon_t$$

Then the observed price changes are:

$$\Delta P_t = \varepsilon_t + \Delta I_t \frac{S}{2}$$

$$E(\Delta P_t) = E(\varepsilon_t) + E(\Delta I_t)\frac{S}{2} = 0$$

$$\text{var}(\Delta P_t) = \text{var}(\varepsilon_t) + \text{var}(\Delta I_t)\frac{S^2}{4} = \sigma^2 + \frac{S^2}{2}$$

$$\begin{aligned}\text{cov}(\Delta P_t, \Delta P_{t-1}) &= E(\Delta P_t \Delta P_{t-1}) \\ &= E\left[\left(\varepsilon_t + \Delta I_t \frac{S}{2}\right)\left(\varepsilon_{t-1} + \Delta I_{t-1}\frac{S}{2}\right)\right] = \frac{S^2}{4}E(\Delta I_t \Delta I_{t-1}) \\ &= \frac{-S^2}{4}\end{aligned}$$

$$\Rightarrow \rho_1 = \frac{-S^2/4}{\sigma^2 + S^2/2} \leq 0$$

Price changes have first order negative autocorrelation even though the change in P* is a fair game.

3. Infrequent or non-synchronous trading

Non-trading can lead to spurious *positive* autocorrelation in stock returns.

Intuition here is that news is incorporated into large caps first and into small caps with a lag/delay (because small caps trade less frequently).

Yesterday's news is present in both yesterday's and today's return via large caps and small caps respectively.

Therefore the returns of an equal weighted index (comprised of small and large caps) will be positively correlated.

Value-weighting can help to mitigate this problem (gives less weight to small caps in the index).

Conclusions

- Information gathering is costly.
- It therefore seems unreasonable to expect the EMH to hold instantly in every period.
  - (Successful) careers in investment management rely on better information acquisition or financial innovation to earn abnormal returns for clients.
- The issue then is one of *relative* efficiency (e.g., How big are the abnormal returns from a particular trading strategy versus a passive buy and hold strategy? Are they long-lasted?)
- Fundamentally, any test of the EMH is conditional on the assumed DGP for prices/returns (joint hypothesis problem).

References

Brooks (2002), Introductory econometrics for finance, CUP: Cambridge.**

Cuthbertson and Nitzsche (2004) Quantitative financial economics: stocks, bonds and foreign exchange, Wiley: Chichester.**

Fama (1991) Efficient capital markets: II, *Journal of Finance*, 46(5), pp1575-1617.

LeRoy (1973) Risk aversion and the martingale property of stock returns, *International Economic Review*, 14, 436-446.

** Key references