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25762 Synthetic Financial Products

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Case Study: A Reset Strike Option

Background

In this case study we are going to investigate an exotic option on a non-dividend-paying stock, known as a *reset strike option*. We begin by introducing some notation:

Symbol	Interpretation
S	Stock price
σ	Stock price volatility
r	Risk-free interest rate (with continuous compounding)
K	Option strike
T	Option maturity
τ	Strike reset date
$C_{\mathrm{R}}(S)$	Price of a reset strike call if the initial stock price is S
$P_{\mathrm{R}}(S)$	Price of a reset strike put if the initial stock price is S
$C(S)$	Price of a vanilla call if the initial stock price is S
$P(S)$	Price of a vanilla put if the initial stock price is S

A reset strike option is a European contingent claim with the feature that its strike price may be modified at the reset date $\tau \in [0, T]$. The way in which the strike price is modified depends on the stock price at the reset date and the flavour of the option. In particular, if the contract is a call, then the reset strike price is $\min\{S_\tau, K\}$. On the other hand, if it is a put, then the reset strike price is $\max\{S_\tau, K\}$. We therefore have the following expressions for the payoffs of the reset strike call and the reset strike put, respectively:

$$\max\Big\{S_T - \min\{S_\tau, K\}, 0\Big\} \qquad \text{and} \qquad \max\Big\{\max\{S_\tau, K\} - S_T, 0\Big\}.$$

Now solve the following problems:

Problems

1. Prove that $C_{\mathrm{R}}(S) \geq C(S)$ and $P_{\mathrm{R}}(S) \geq P(S)$, for all values S.
2. For what value of τ do the inequalities above become equalities?
3. Using risk-neutral pricing, it is possible to prove that

$$\begin{aligned} C_{\mathrm{R}}(S) = {} & S\mathcal{M}(a_1, y_1, \rho) - Ke^{-rT}\mathcal{M}(a_2, y_2, \rho) - Se^{-r(T-\tau)}\mathcal{N}(-a_1)\mathcal{N}(z_2) \\ & + S\mathcal{N}(-a_1)\mathcal{N}(z_1) \end{aligned}$$

and

$$P_{\mathrm{R}}(S) = Se^{-r(T-\tau)}\mathcal{N}(a_1)\mathcal{N}(-z_2) - S\mathcal{N}(a_1)\mathcal{N}(-z_1) + Ke^{-rT}\mathcal{M}(-a_2,-y_2,\rho) \\ - S\mathcal{M}(-a_1,-y_1,\rho),$$

where

$$a_1 := \frac{\ln(S/K)+(r+\sigma^2/2)\tau}{\sigma\sqrt{\tau}}; \qquad a_2 := a_1 - \sigma\sqrt{\tau};$$
$$z_1 := \frac{(r+\sigma^2/2)(T-\tau)}{\sigma\sqrt{T-\tau}}; \qquad a_2 := a_1 - \sigma\sqrt{T-\tau};$$
$$y_1 := \frac{\ln(S/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}; \qquad y_2 := a_1 - \sigma\sqrt{T};$$

and

$$\rho := \sqrt{\tau/T}.$$

In the expressions above,

$$\mathcal{N}(x) := \mathsf{P}(Z \le x)$$

is the cumulative distribution function of a standard normal random variable Z, while

$$\mathcal{M}(x_1,x_2,\rho) := \mathsf{P}(Z_1 \le x_1, Z_2 \le x_2)$$

is the joint cumulative distribution function of two standard normal random variables Z_1 and Z_2, whose correlation is ρ. The *Excel* workbook `Reset Strike Option.xls`, which can be downloaded from *UTSOnline*, contains an implementation of this function.[1] In particular, we have

$$\texttt{bivar}(x_1,x_2,\rho) := \mathcal{M}(x_1,x_2,\rho).$$

Now, using the parameter values $r = 5\%$, $\sigma = 30\%$, $K = 100$, $T = 1$ year and $\tau = 6$ months, plot the pricing functions of the reset strike call and the reset strike put against the stock price, for $S \in [50, 150]$.

4. Use the data $r = 5\%$, $\sigma = 30\%$, $S = 100$, $K = 100$, $T = 1$ year and $\tau = 6$ months, to price the reset strike call and the reset strike put with a 12-step binomial tree.[2] Explain the methodology you used in detail. (**Hint:** You have to be careful here, since reset strike options are path-dependent. This means that the payoff of such an instrument at a terminal node of the binomial tree does not depend exclusively on the stock price at the node—it also depends on the path followed by the stock price in order to get there.)
5. The risk-neutral distribution of the stock price at times τ and T are determined by

$$S_\tau \sim S_0 e^{(r-\frac{1}{2}\sigma^2)\tau+\sigma\sqrt{\tau}Z_1} \qquad \text{and} \qquad S_T \sim S_\tau e^{(r-\frac{1}{2}\sigma^2)(T-\tau)+\sigma\sqrt{T-\tau}Z_2},$$

respectively, where Z_1 and Z_2 are independent standard normal random variables. Using the parameter values $S_0 = 100$, $r = 5\%$ and $\sigma = 30\%$, generate $N = 1000$ samples of S_τ, which we denote by $\hat{S}_\tau^{(1)},\ldots,\hat{S}_\tau^{(N)}$. In order to do this, use the *Analysis Toolpack* to generate N samples of Z_1, which we denote by $\hat{Z}_1^{(1)},\ldots,\hat{Z}_1^{(N)}$. Next, generate N samples of Z_2, denoted by $\hat{Z}_2^{(1)},\ldots,\hat{Z}_2^{(N)}$, which you should

[1] When you download this workbook, make sure that you enable its macros.

[2] In other words, each time-step of the binomial tree should be $\Delta t = 1$ month.

use in conjunction with the sample values of S_τ to generate N sample values of S_T, which we denote by $\hat{S}_T^{(1)}, \ldots, \hat{S}_T^{(N)}$. Plot the histograms of your sample distributions of S_τ and S_T.

6. Use the data $r = 5\%$, $\sigma = 30\%$, $S = 100$, $K = 100$, $T = 1$ year and $\tau = 6$ months, as well as the sample values of S_τ and S_T you generated for Problem 5, to price the reset strike call and the reset strike put by Monte Carlo integration. In detail, we obtain the following Monte Carlo estimates for the prices of these options:

$$\hat{C}_\mathrm{R} \approx e^{-rT} \frac{1}{N} \sum_{i=1}^{N} \max\Big(\hat{S}_T^{(i)} - \min\big(\hat{S}_\tau^{(i)}, K\big), 0 \Big)$$

and

$$\hat{P}_\mathrm{R} \approx e^{-rT} \frac{1}{N} \sum_{i=1}^{N} \max\Big(\max\big(\hat{S}_\tau^{(i)}, K\big) - \hat{S}_T^{(i)}, 0 \Big).$$

(Monte Carlo simulation is discussed in Chapter 19 of your textbook.)